

ST MARY'S CHURCH OF ENGLAND PRIMARY SCHOOL

Year 1 - 6

Calculation Policy

Addition and Subtraction



St Mary's Church of England Primary School Calculation Policy adapted from White Rose Maths, and NCETM professional development materials.

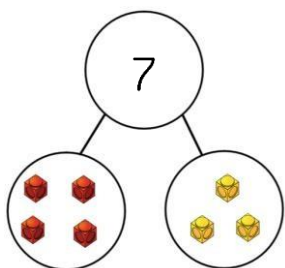
This document is broken down into addition and subtraction, and multiplication and division.

At the start of each policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

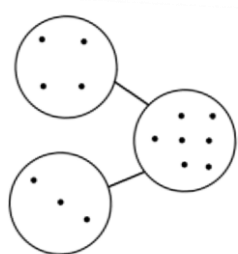
There is an overview of the skills linked to year groups to support consistency throughout school. A glossary of terms is provided at the end of the calculation policy to support the understanding of the key language used to teach the four operations.

Part-Whole Model



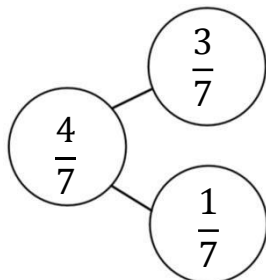
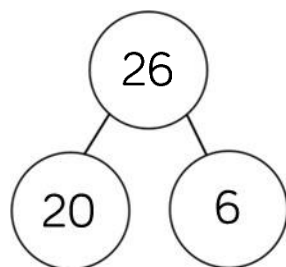
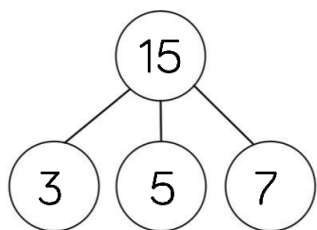
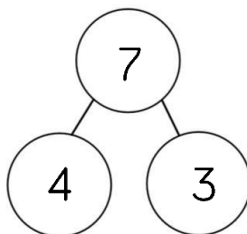
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

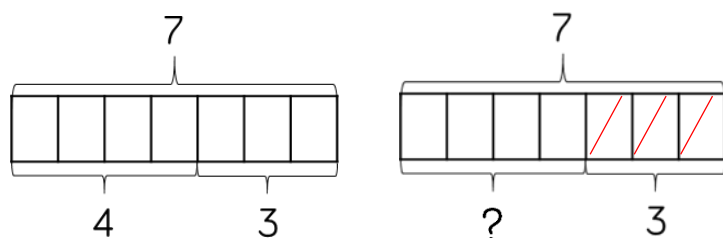
In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Part-Whole Model

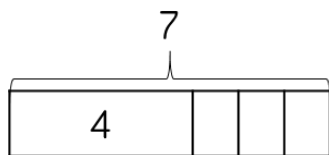
Concrete



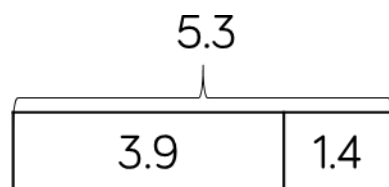
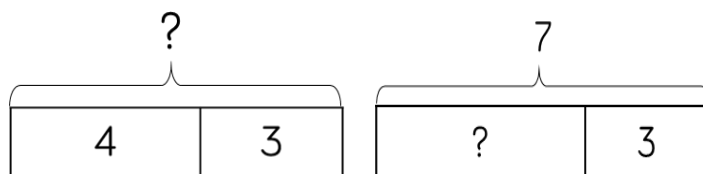
Discrete



Combination



Continuous



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

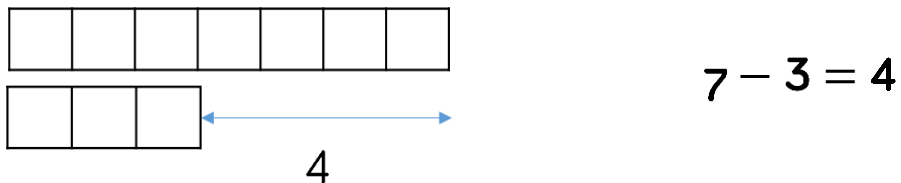
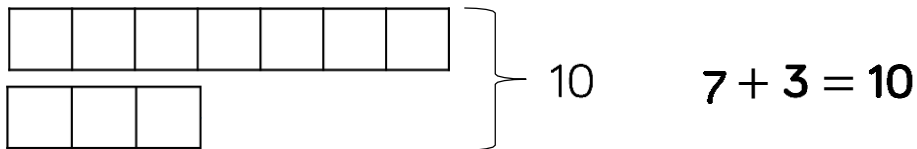
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

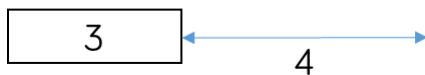
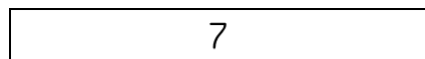
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Part-Whole Model

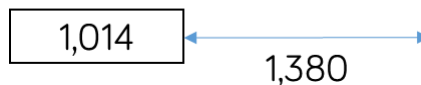
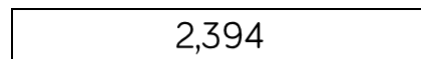
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



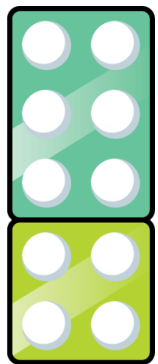
$$7 = 4 + 3$$



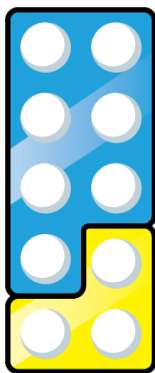
$$7 = 3 + 4$$



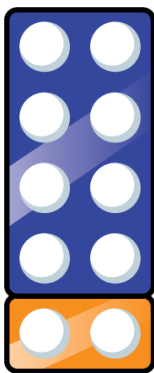
$$7 - 3 = 4$$



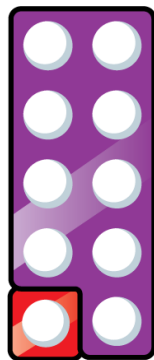
$$6 + 4$$



$$7 + 3$$



$$8 + 2$$



$$9 + 1$$

Benefits

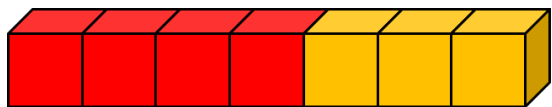
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

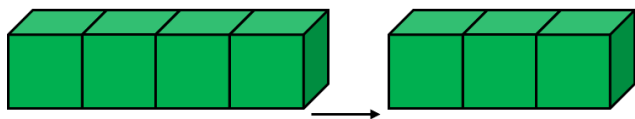
Number Shapes



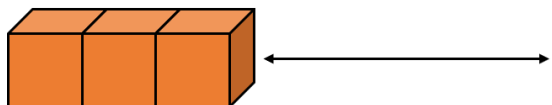
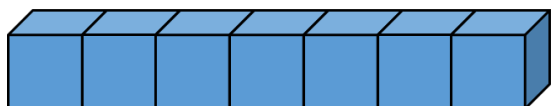
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

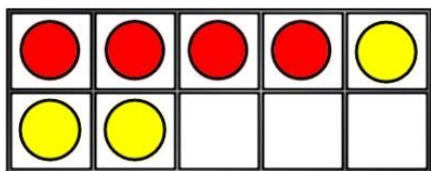
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Number Shapes



$$4 + 3 = 7$$

$$3 + 4 = 7$$

$$7 - 3 = 4$$

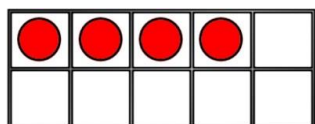
$$7 - 4 = 3$$

4 is a part.

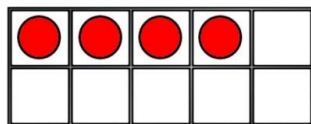
3 is a part.

7 is the whole.

First

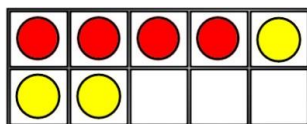


Then

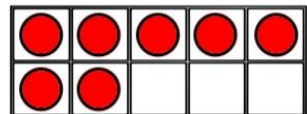


$$4 + 3 = 7$$

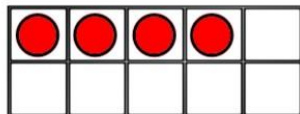
Now



First

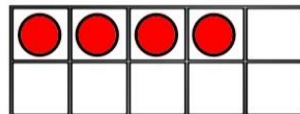


Then



$$7 - 3 = 4$$

Now



Benefits

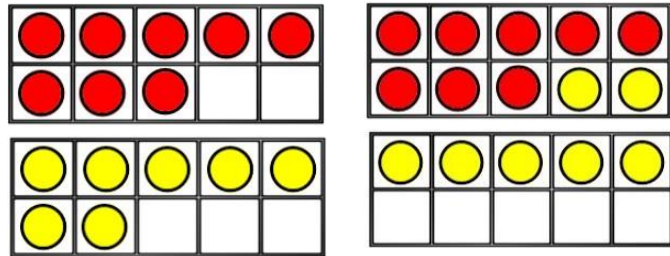
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

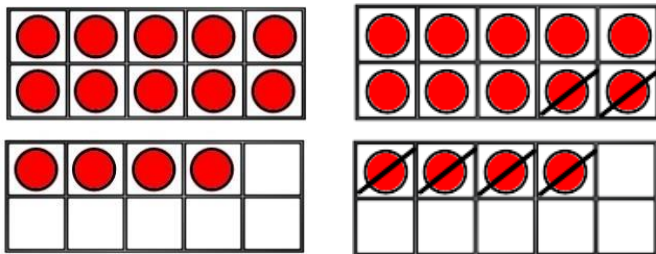
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

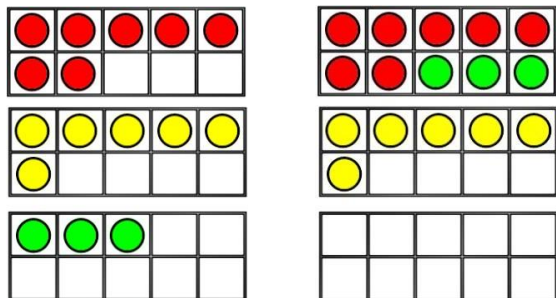
Ten Frames (within 20)



$$\begin{array}{r} 8 + 7 = 15 \\ \quad \swarrow \searrow \\ 2 \quad 5 \end{array}$$



$$\begin{array}{r} 14 - 6 = 8 \\ \quad \swarrow \searrow \\ 4 \quad 2 \end{array}$$



$$\begin{array}{r} 7 + 6 + 3 = 16 \\ \quad \swarrow \quad \searrow \\ \quad 10 \end{array}$$

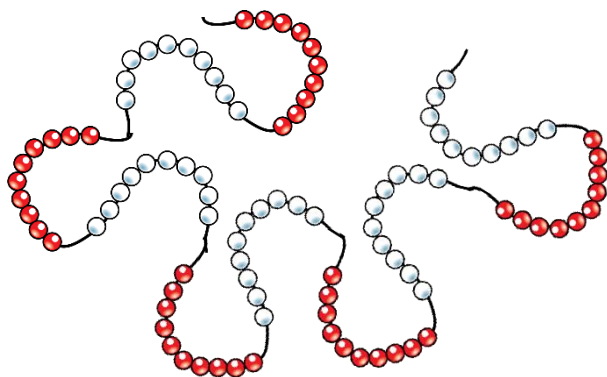
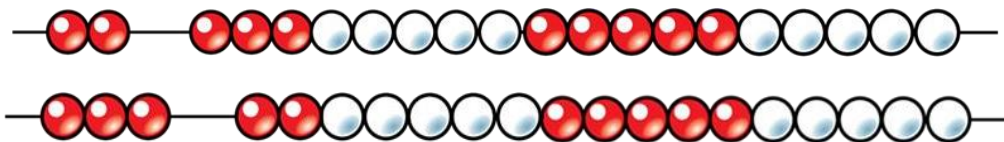
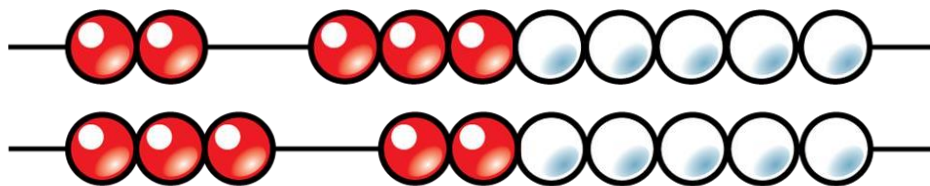
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Ten Frames (within 20)



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

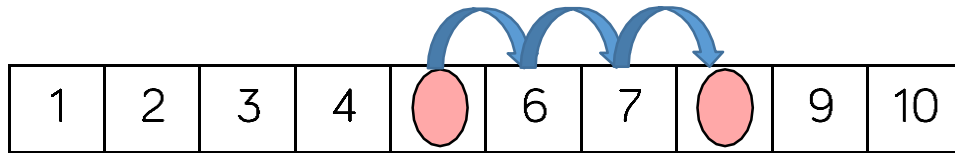
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

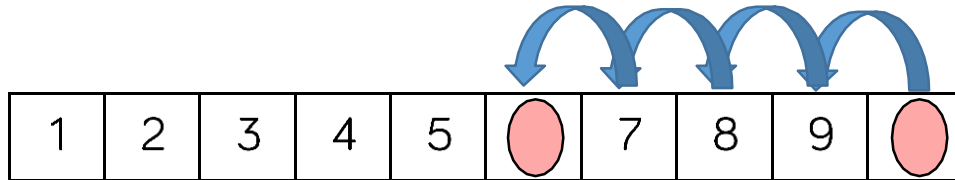
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Ten Frames (within 20)

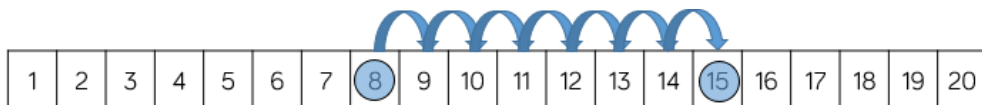
$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

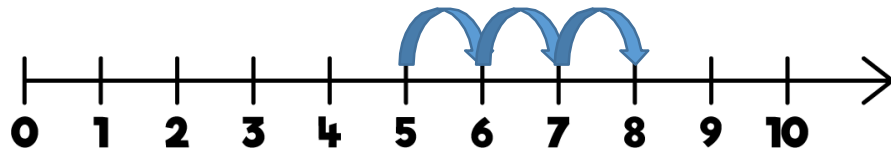
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

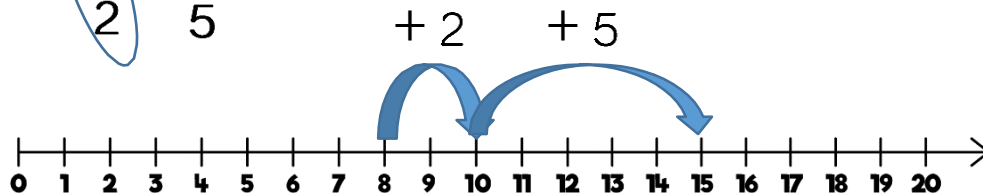
Ten Frames (within 20)

$$5 + 3 = 8$$



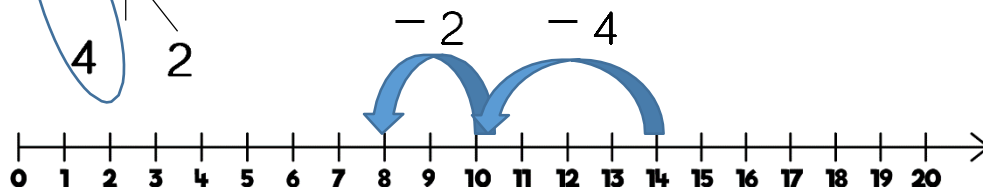
$$8 + 7 = 15$$

The number 7 is partitioned into 2 and 5. The number 8 is circled in blue.



$$14 - 6 = 8$$

The number 6 is partitioned into 4 and 2. The number 14 is circled in blue.



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

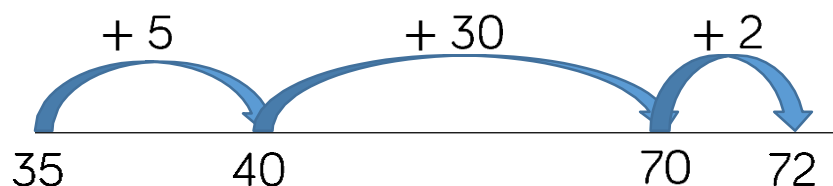
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

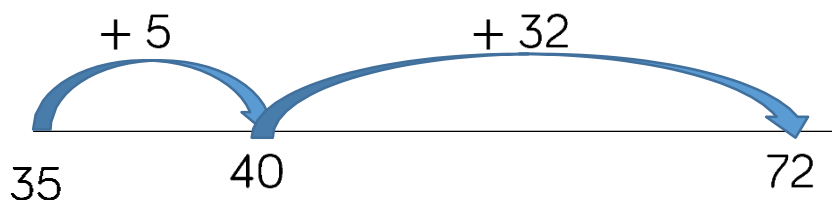
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Ten Frames (within 20)

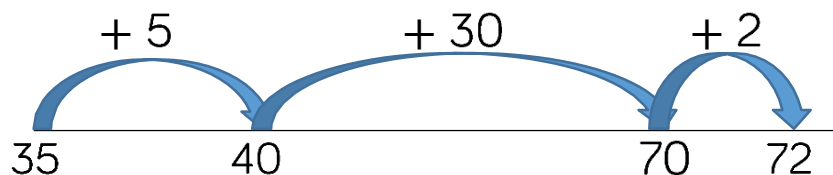
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

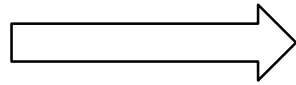
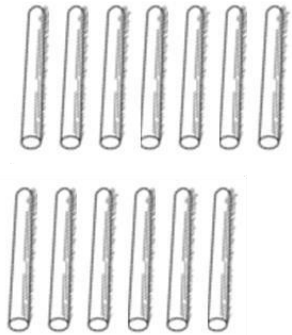
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

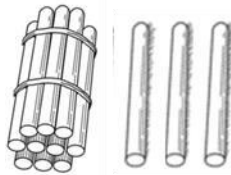
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Ten Frames (within 20)

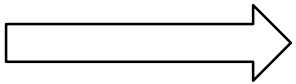
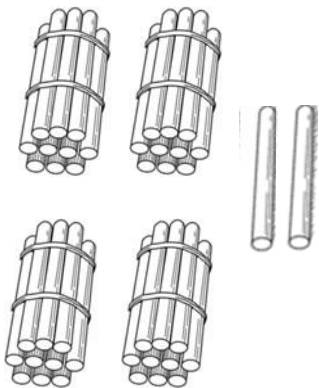
$$7 + 6 = 13$$



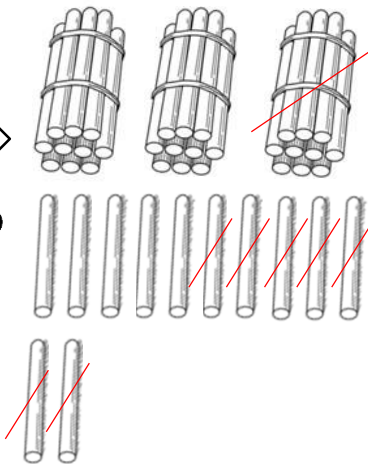
bundle together
groups of 10



$$42 - 17 = 25$$



unbundle group
of 10 straws



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

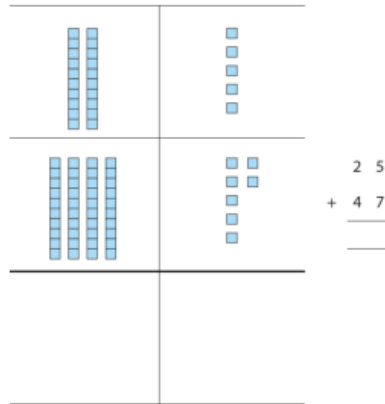
When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

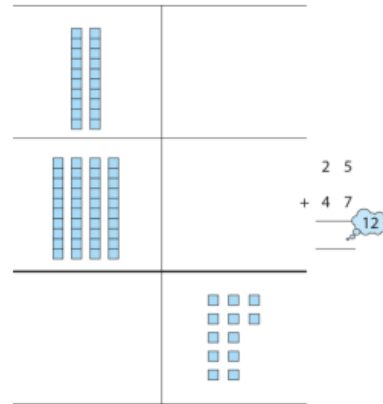
Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Ten Frames (within 20)

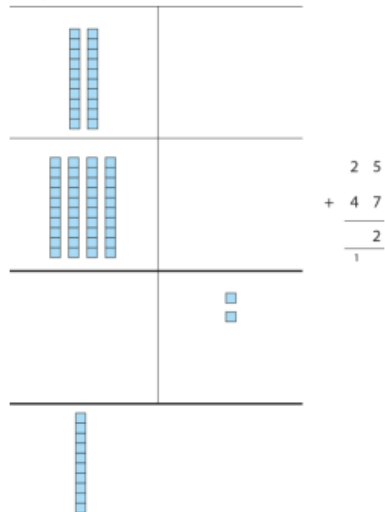
Step 1



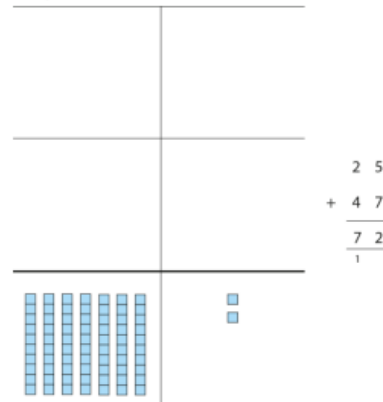
Step 2



Step 3



Step 4



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

How many ones are there altogether?

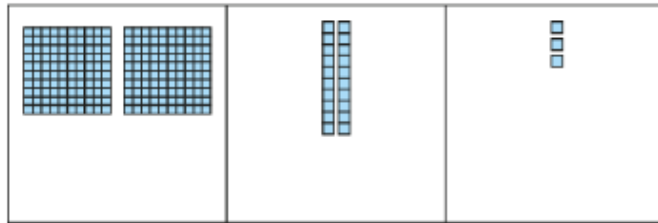
Can we make an exchange? (Yes or No)

How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)

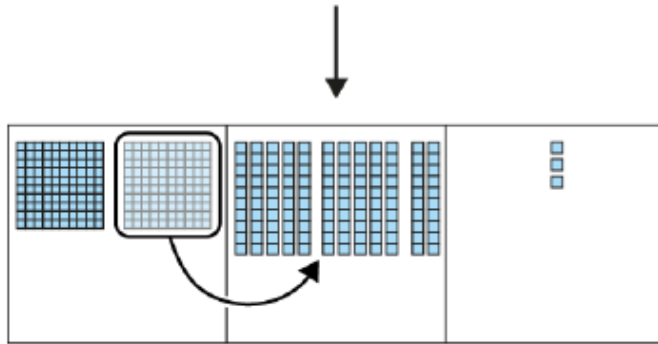
How many ones do we have left? (Write in ones column)

Repeat for each column.

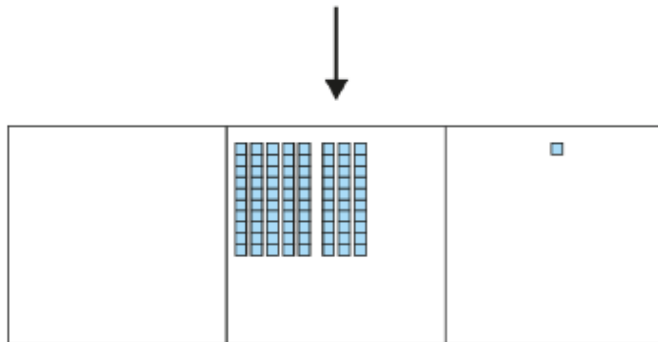
Ten Frames (within 20)



100s	10s	1s
2	2	3
- 1	4	2



100s	10s	1s
2 ¹	¹ 2	3
- 1	4	2



100s	10s	1s
2 ¹	¹ 2	3
- 1	4	2
0	8	1

Benefits

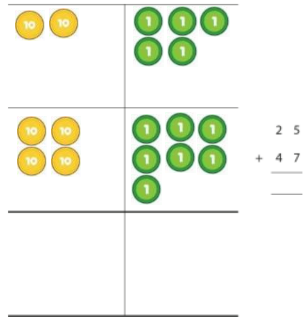
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

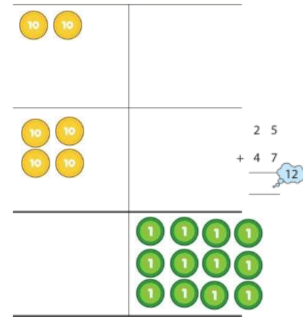
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Ten Frames (within 20)

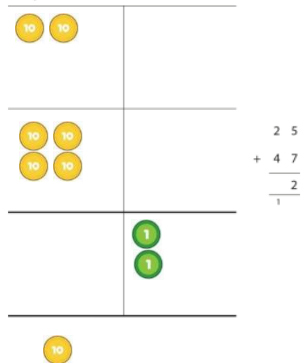
Step 1



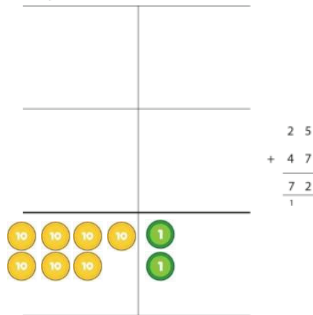
Step 2



Step 3



Step 4



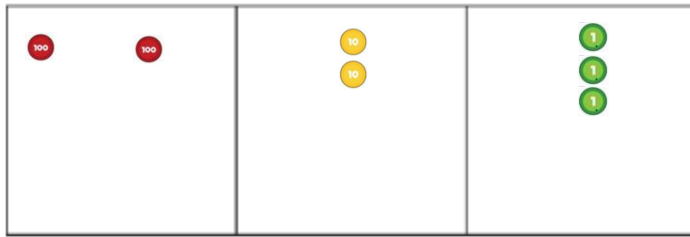
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

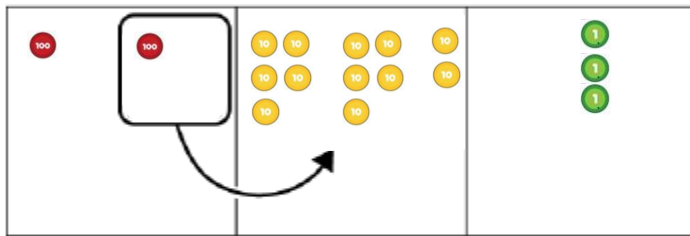
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

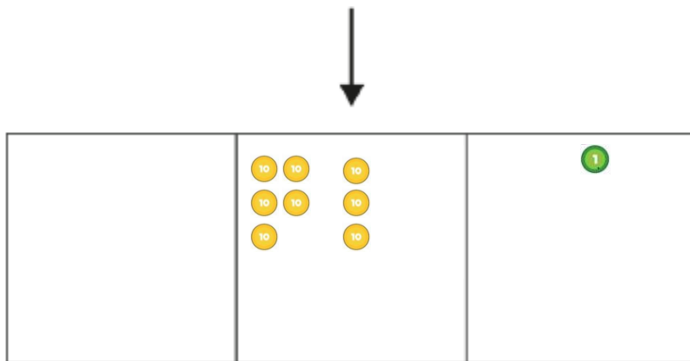
Place Value Counters (Subtraction)



100s	10s	1s
2	2	3
- 1	4	2



100s	10s	1s
2 ¹	12	3
- 1	4	2



100s	10s	1s
2 ¹	12	3
- 1	4	2
0	8	1

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

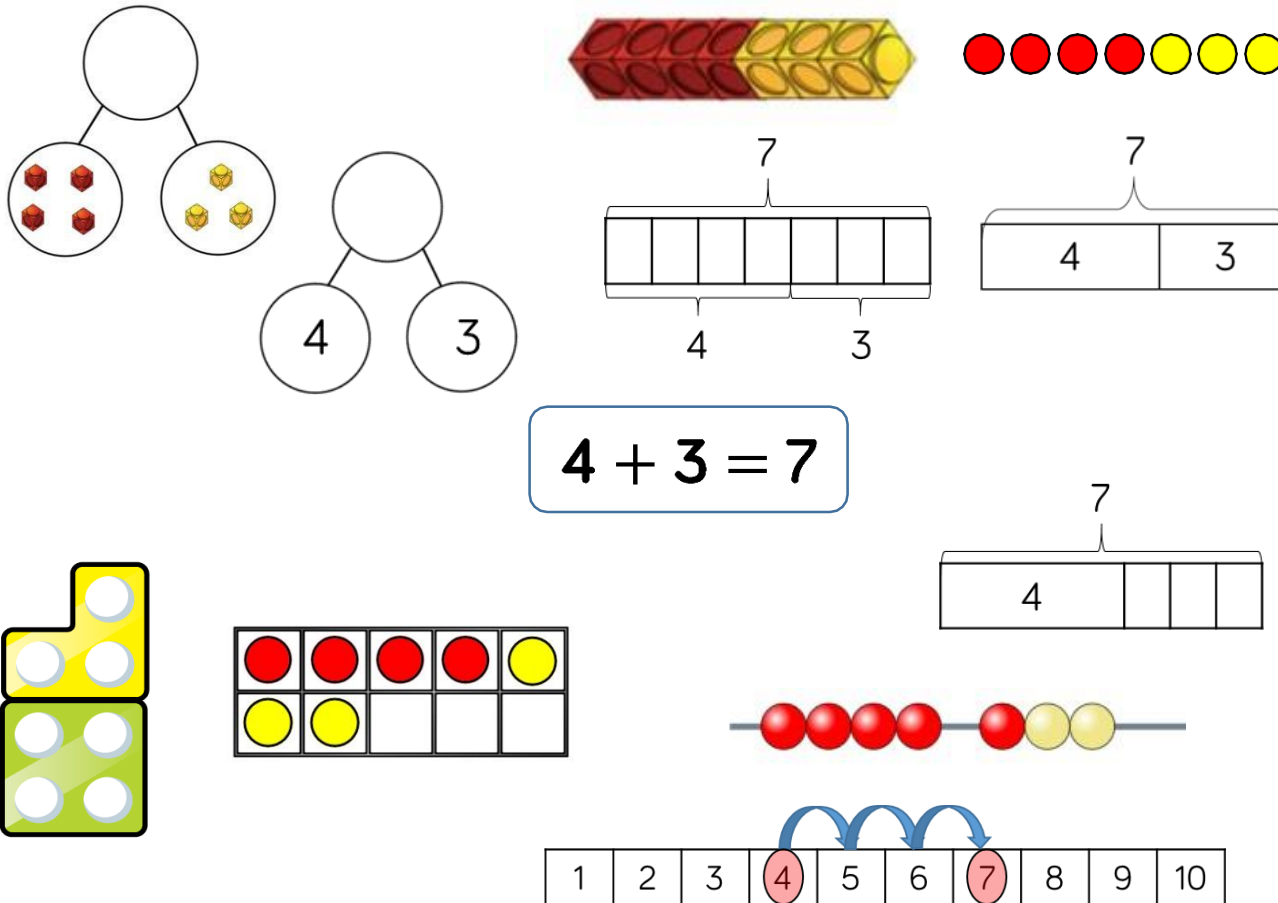
Addition

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square

Skill	Year	Representations and models	
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters
Add with up to 3-digits	3	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters Column addition
Add with more than 4 digits	5	Part-whole model Bar model Number lines (blank)	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model Number lines (blank)	Place value counters Column addition

Skill: Add 1-digit numbers within 10

Year: 1



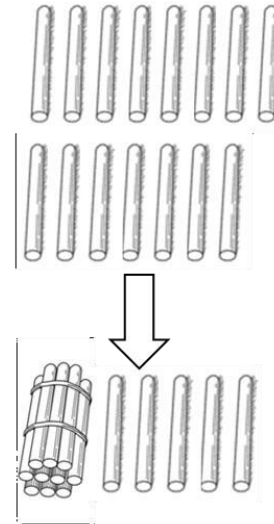
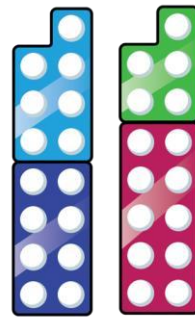
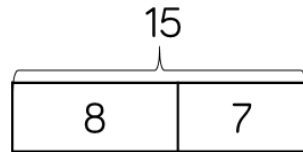
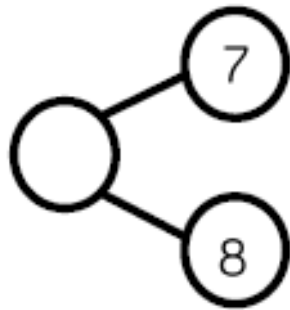
When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add 1 and 2-digit numbers to 20

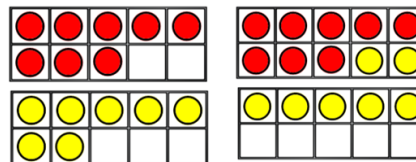
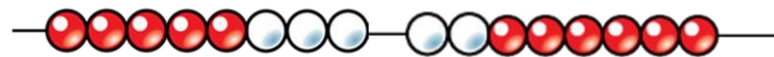
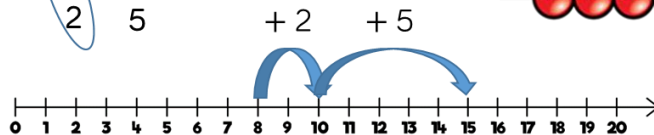
Year: 1/2



$$8 + 7 = 15$$

$$8 + 7 = 15$$

2 5



$$8 + 7 = 15$$

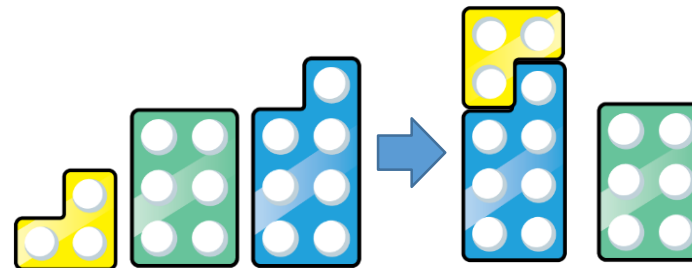
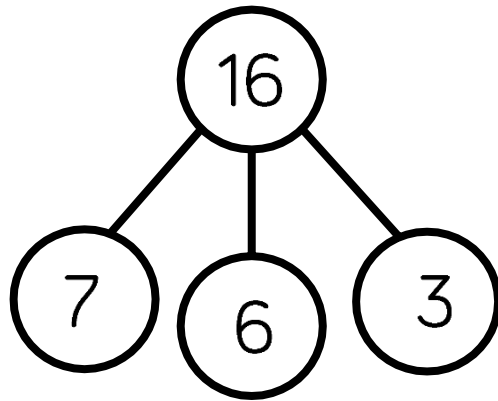
2 5

When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

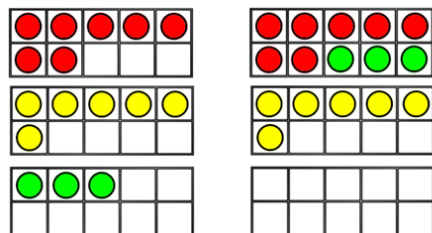
Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

Skill: Add three 1-digit numbers

Year: 2

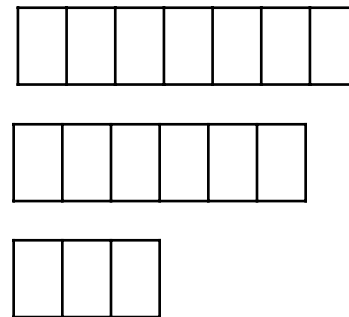


$$7 + 6 + 3 = 16$$



$$7 + 6 + 3 = 16$$

10



16

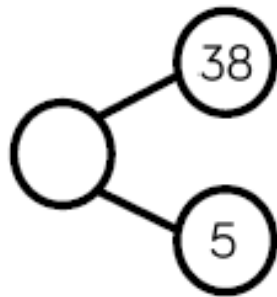
When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

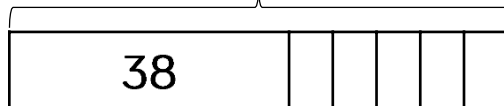
Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

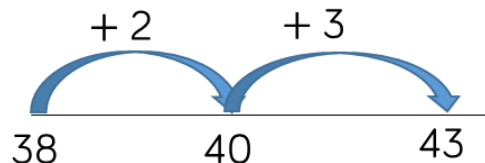
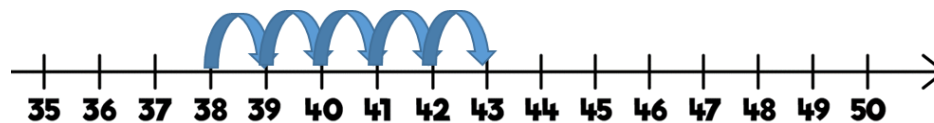
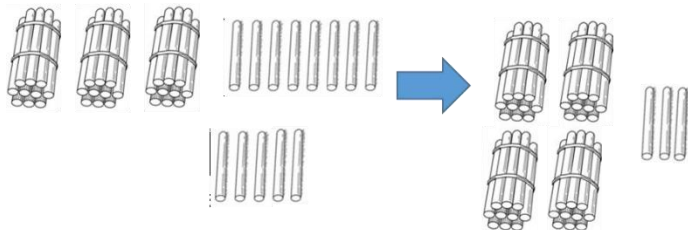
Year: 2/3



?



$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

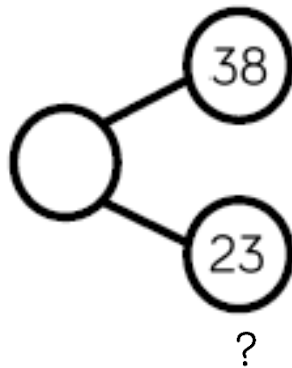
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

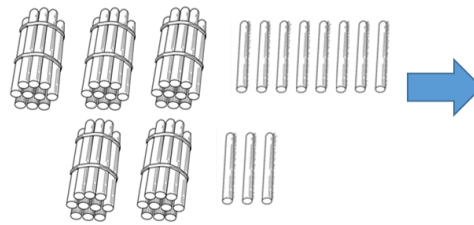
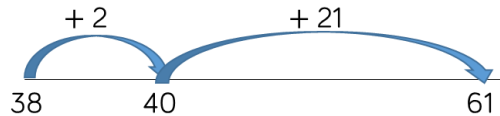
Hundred squares and straws can support children to find the number bond to 10.

Skill: Add two 2-digit numbers to 100

Year: 2/3



38	23
----	----



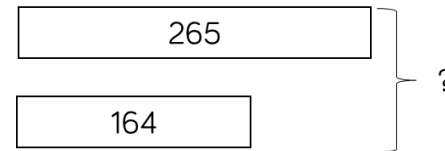
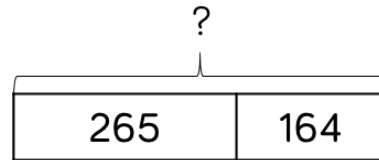
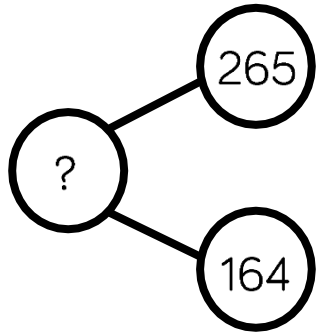
$$38 + 23 = 61$$

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

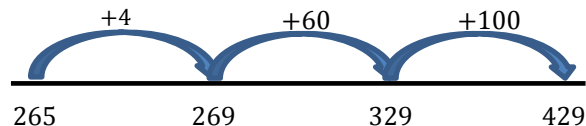
Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Add numbers with up to 3 digits

Year: 3



$$265 + 164 = 429$$



Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

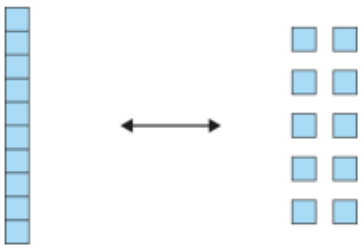
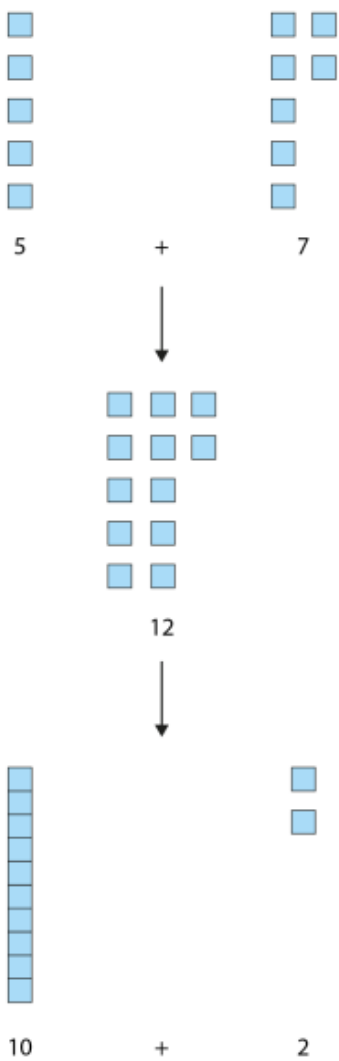
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Teaching point 4:

If any column sums to ten or greater, we must 'regroup'.

Steps in learning

	Guidance	Representations
4:1	It is probable that children will take longer to master this teaching point than the previous ones in the segment. Begin by reviewing and extending previous work on grouping ten ones into one ten using manipulatives, such as Dienes.	
4:2	Spend some time performing addition calculations, and then regrouping, outside of the context of column addition, using Dienes. Encourage children to describe the process in full sentences using the language of unitising: • 'Five ones plus seven ones is equal to twelve ones.' • 'Twelve ones is equal to one ten and two ones.'	

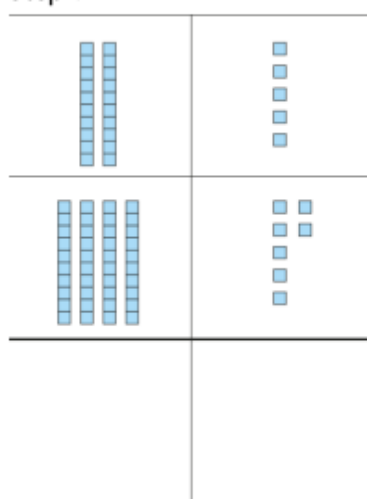
4:3

Once it is clear that children have mastered the previous step, you can begin applying the column addition algorithm with regrouping in the ones column, demonstrating a consistent approach for recording the regrouping. You may wish to begin with an example in which the ones digits sum to exactly ten, before moving onto more general calculations with regrouping.

As in *Teaching point 3*, initially use Dienes alongside the abstract representation to help bridge the steps between concepts, and continue to use the language of unitising in ones and tens.

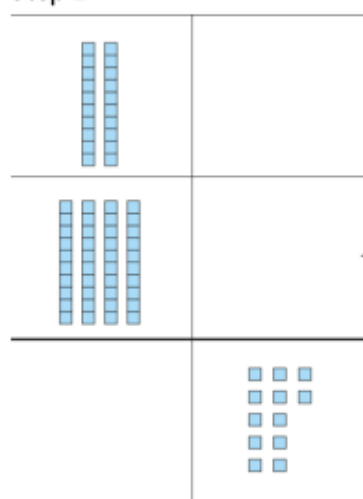
Encourage children to reason *why* a total of ten in the ones column should be exchanged for one ten and placed in the tens column, referring back to the previous steps. Work towards use of the generalised statement: **'If the column sum is equal to ten or more, we must regroup.'**

Step 1



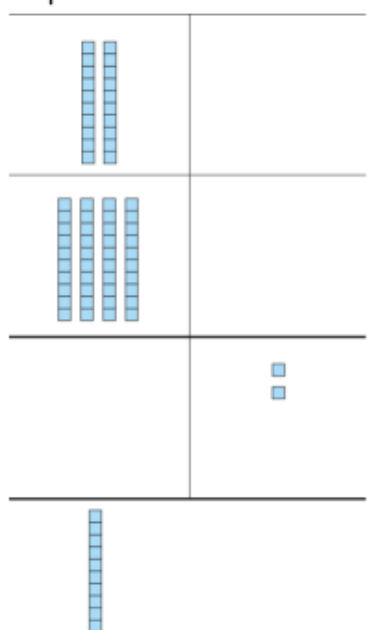
$$\begin{array}{r} 25 \\ + 47 \\ \hline \end{array}$$

Step 2



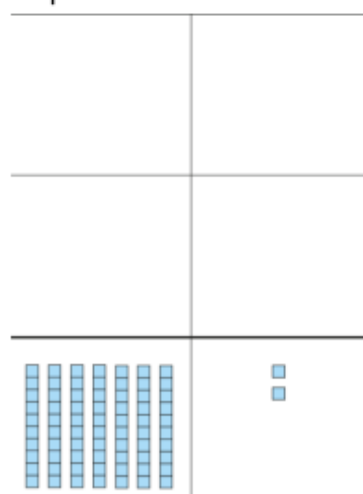
$$\begin{array}{r} 25 \\ + 47 \\ \hline 12 \end{array}$$

Step 3



$$\begin{array}{r} 25 \\ + 47 \\ \hline 2 \end{array}$$

Step 4



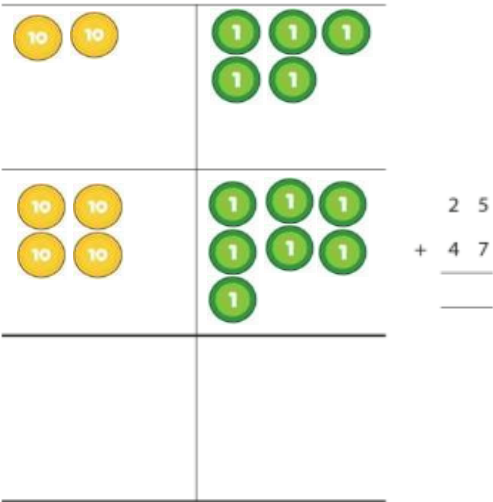
$$\begin{array}{r} 25 \\ + 47 \\ \hline 72 \end{array}$$

4:4

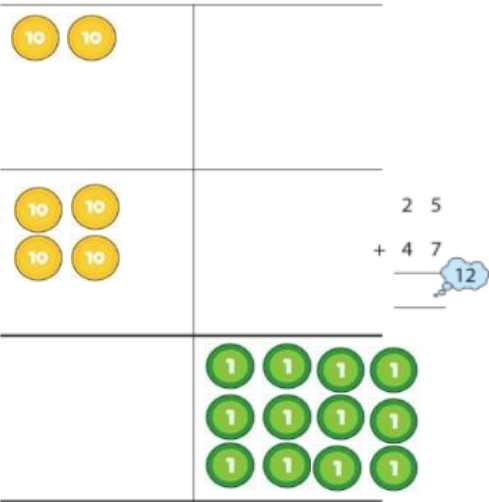
Repeat steps 4:1–4:3 for regrouping ten tens into one hundred.

Column Addition (Place Value Counters)

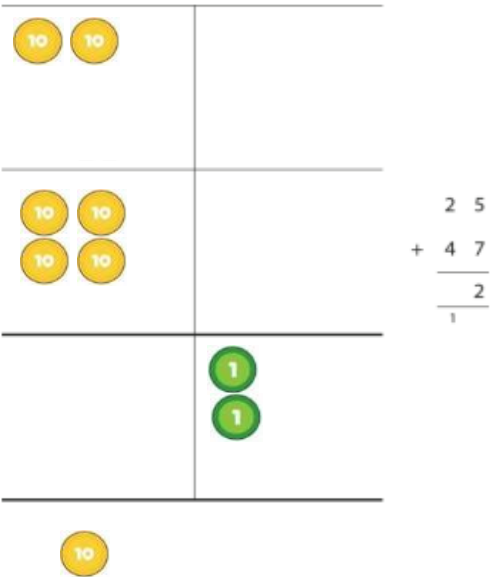
Step 1



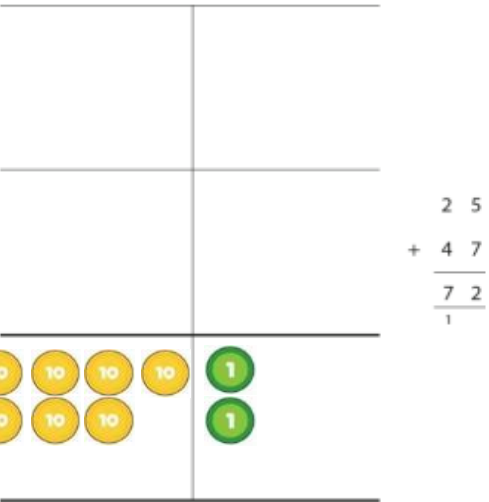
Step 2



Step 3

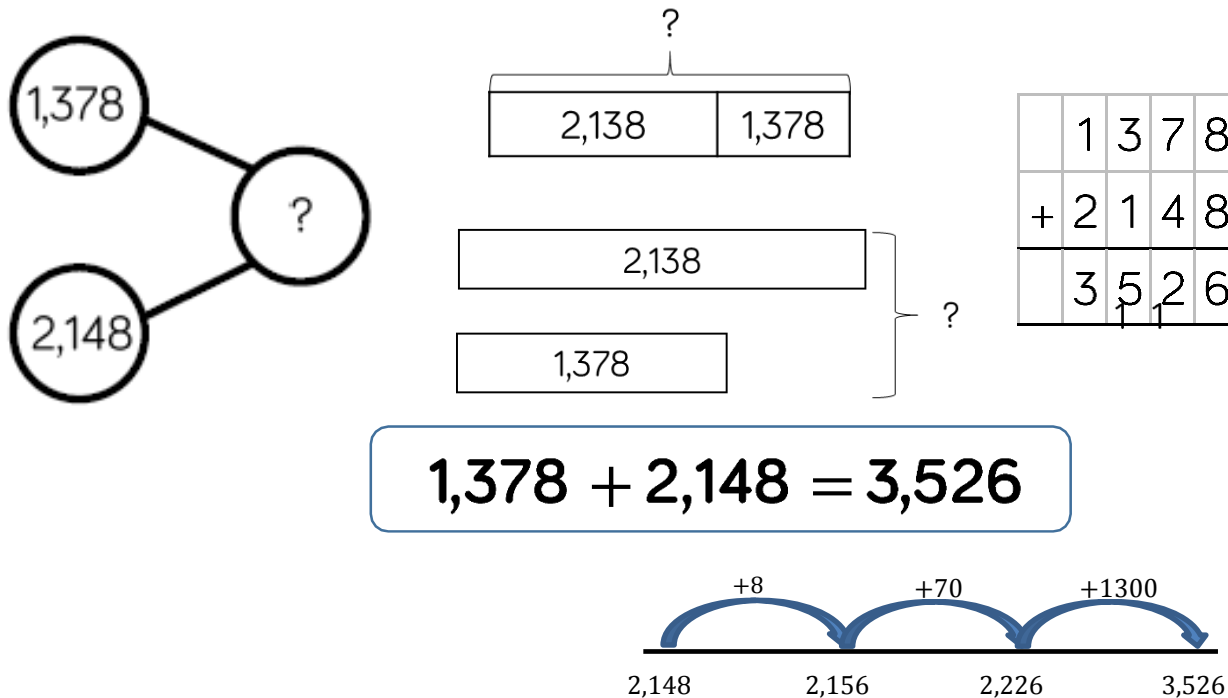


Step 4



Skill: Add numbers with up to 4 digits

Year: 4



Refer to Year 3 column addition method and apply with numbers up to 4 digits.
Ensure Base 10 is used before moving onto using place value counters.

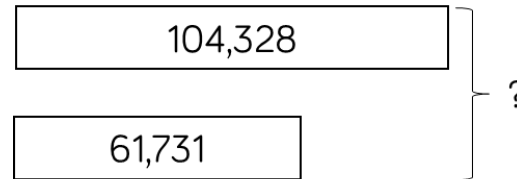
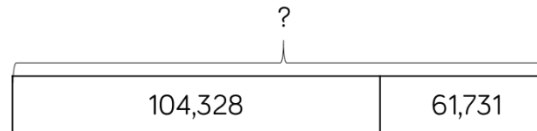
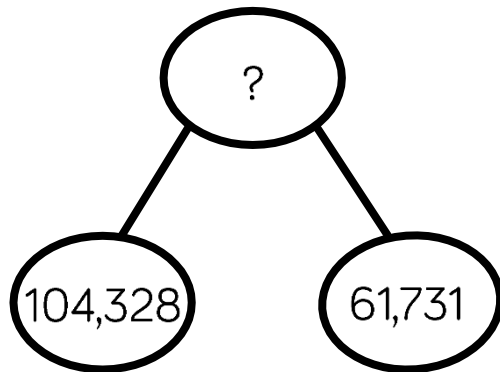
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

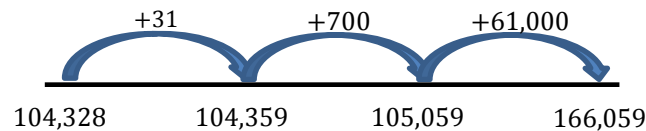
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

Year: 5/6



$$104,328 + 61,731 = 166,059$$



1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

1

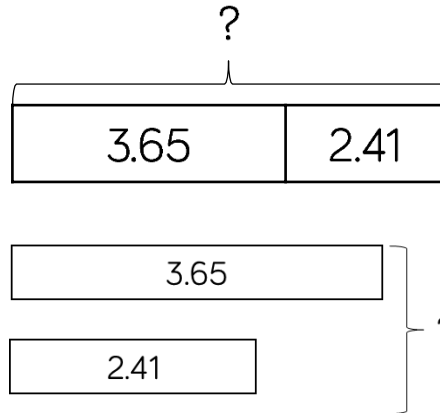
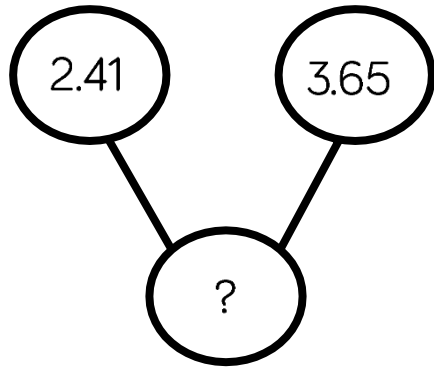
Refer to Year 3 column addition method and apply with numbers greater than 4 digits.

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Skill: Add with up to 3 decimal places

Year: 5

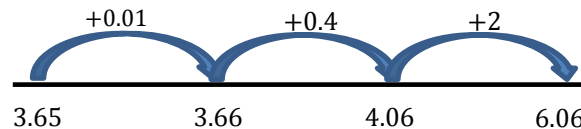


$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

$$3.65 + 2.41 = 6.06$$

Refer to Year 3 column addition method and apply with decimals.

Base 10 to be used before place value counters to develop children's understanding of decimals.



Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

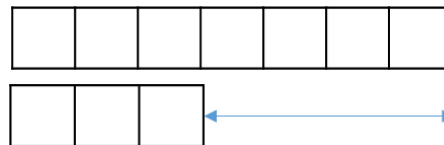
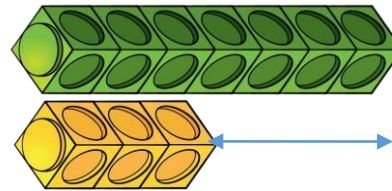
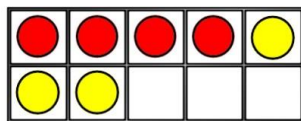
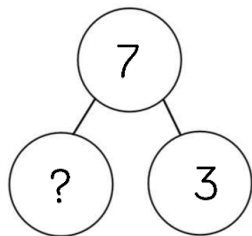
Subtraction

Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model	Ten frames (within 10) Bead strings (10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Ten frames (within 20)	Bead string (20) Number tracks Number lines (labelled) Straws
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters

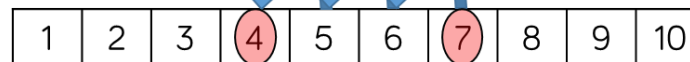
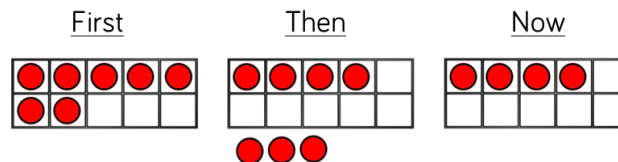
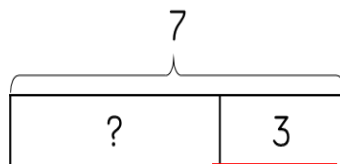
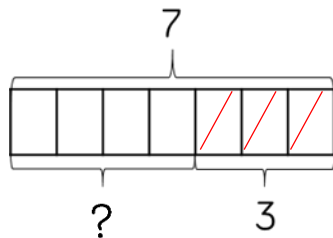
Skill	Year	Representations and models	
Subtract with up to 3-digits	3	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters Column addition
Subtract with up to 4-digits	4	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters Column addition
Subtract with more than 4 digits	5	Part-whole model Bar model Number lines (blank)	Place value counters Column addition
Subtract with up to 3 decimal places	5	Part-whole model Bar model Number lines (blank)	Place value counters Column addition

Skill: Subtract 1-digit numbers within 10

Year: 1



$$7 - 3 = 4$$



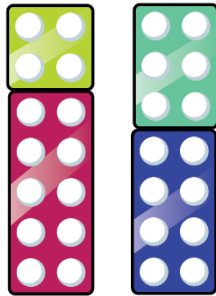
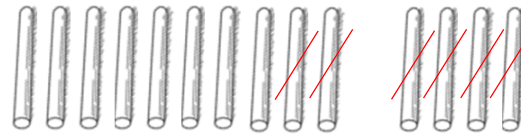
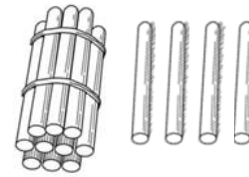
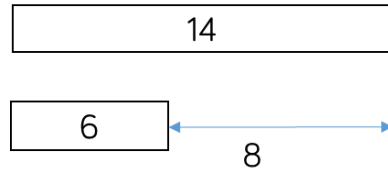
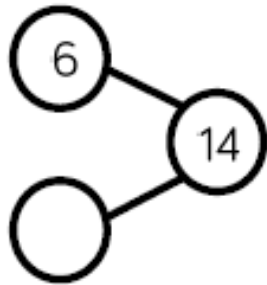
Part-whole models, bar models, ten frames and number shapes support partitioning.

Ten frames, number tracks, single bar models and bead strings support reduction.

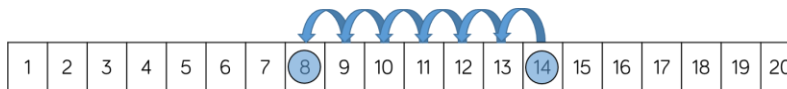
Cubes and bar models with two bars can support finding the difference.

Skill: Subtract 1 and 2-digit numbers to 20

Year: 1/2

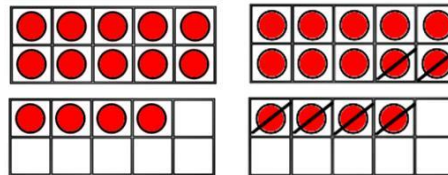
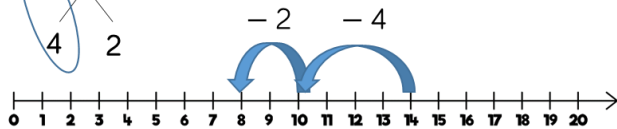


$$14 - 6 = 8$$



$$14 - 6 = 8$$

A diagram showing the number 14 with a bracket underneath it. The bracket is divided into two parts: the left part is labeled '4' and the right part is labeled '2'.



$$14 - 6 = 8$$

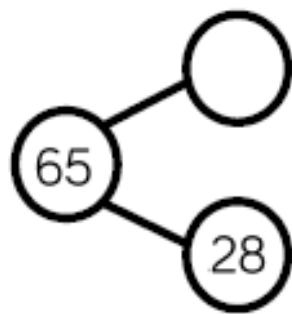
A diagram showing the number 14 with a bracket underneath it. The bracket is divided into two parts: the left part is labeled '4' and the right part is labeled '2'.

When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

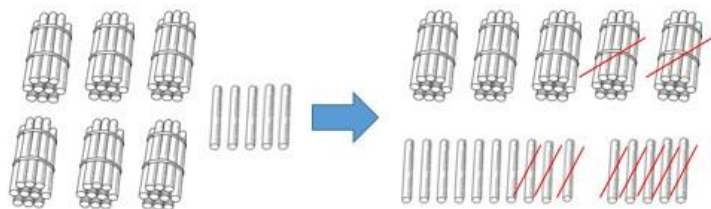
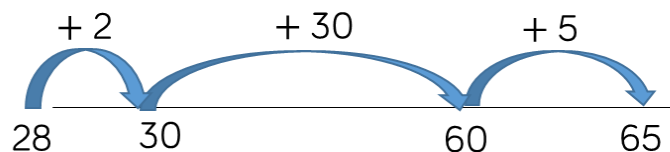
Skill: Subtract 1 and 2-digit numbers to 100

Year: 2



65

65	
?	28



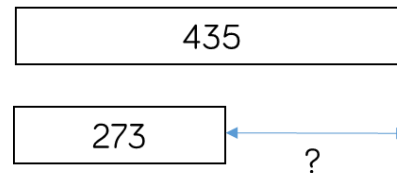
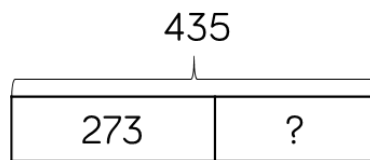
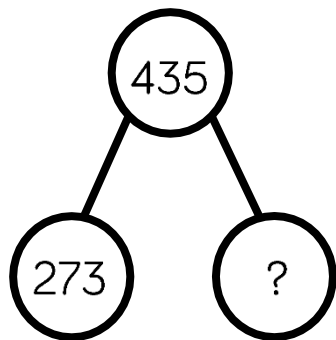
$$65 - 28 = 37$$

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

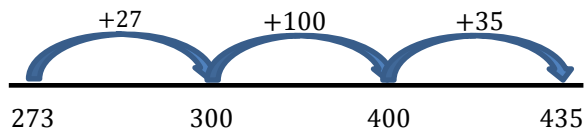
Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits

Year: 3



$$435 - 273 = 262$$



Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Teaching point 2:

If there is an insufficient number of any unit to subtract from in a given column, we must exchange from the column to the left.

Steps in learning

	Guidance	Representations
2:1	Begin by reviewing the partitioning of two-digit numbers into tens and ones, including exchanging one of the tens for ten ones (see segment 1.14 <i>Addition and subtraction: two-digit numbers and multiples of ten</i>, Teaching point 4 for different ways of partitioning two-digit numbers). Children should understand, for example, that 23 is equivalent to two tens and three ones as well as one ten and thirteen ones. Use Dienes and part-part-whole diagrams as shown opposite, continuing to use the language of unitising.	
2:2	You could introduce the idea of exchange in column subtraction using the following game: • Provide each child with: • a set of 99 in Dienes (nine tens and nine ones) on a chart (without place-value headings) • spare ones Dienes • Provide each pair of children with a six-sided- or nine-sided-die. • Working in pairs, child A rolls the die and then subtracts the result from his/her own Dienes. Child B checks the calculation has been performed correctly. • Child B then takes a turn rolling the die and subtracting the value from his/her Dienes.	

	- Initially children will subtract from the ones; for example, Megan rolls a five on her first turn ($99 - 5 = 94$). However, they will eventually need to exchange a ten for ten ones in order to subtract the correct amount; for example, Megan then rolls a six on her second turn, as shown opposite ($94 - 6 = 88$). After exchanging one Dienes ten rod with ten ones, encourage children to move the ten ones over to the column with the rest of the ones. - The children continue taking turns until one child runs out of Dienes; the child who is the last left with Dienes on their chart wins.	Example turn: $94 - 6$ The diagram illustrates the process of subtracting 6 from 94 using Dienes blocks. It is divided into four stages connected by downward arrows. Stage 1 shows 9 tens rods and 4 ones units. Stage 2 shows one ten rod being exchanged for ten ones units, resulting in 8 tens rods and 14 ones units. Stage 3 shows 6 ones units being removed from the ones column, leaving 8 tens rods and 8 ones units. Stage 4 shows the final result: 8 tens rods and 8 ones units.
2:3	Now introduce the abstract representation alongside the practical or pictorial. Demonstrate some example turns of the game described in step 2:2, working through the corresponding calculations with children. Then children can play the game again using both Dienes <i>and</i> the column subtraction representation. The Dienes should be used to help children understand the underlying structure of exchange, rather than as a tool for calculation. Make sure that children can relate the reduction of the tens value with the introduction of an additional '1' in the ones column of the algorithm. Children should then	

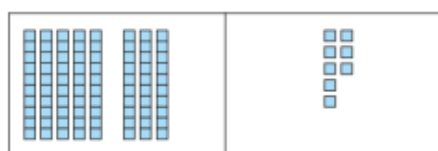
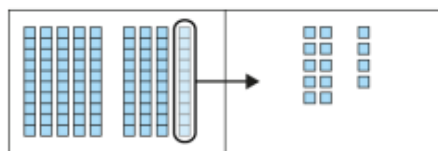
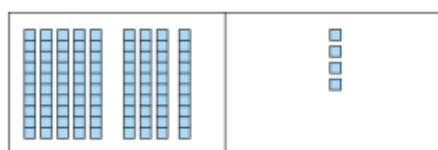
1.21 Column subtraction

perform the subtraction in each column using known facts.

After a short period of practice in pairs, remove the Dienes scaffold to ensure that children can work with only the abstract representation; similarly, fairly quickly remove the scaffold of the place-value headings in the column algorithm.

Example turn with Dienes and column subtraction:

$$94 - 6$$



10s	1s
9	4
-	6

10s	1s
9 ⁸	14
-	6

10s	1s
9 ⁸	14
-	6
8	8

2:4

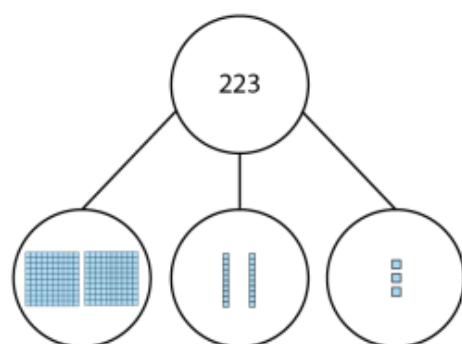
When children have mastered exchanging tens for ones, progress to calculations with three-digit numbers, exploring exchange of one hundred with ten tens.

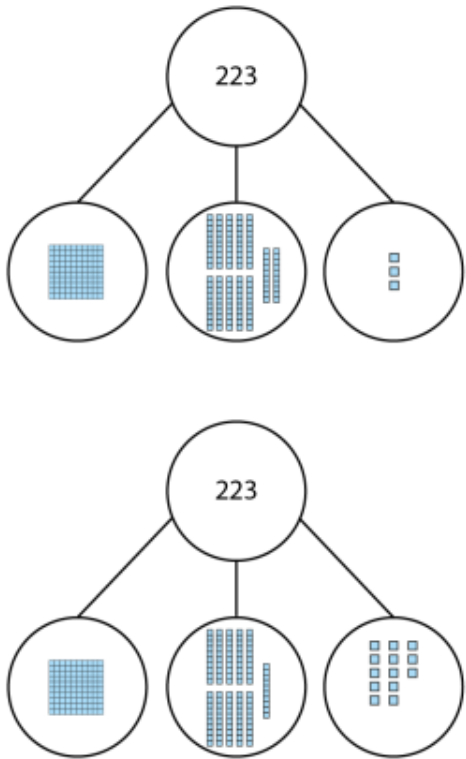
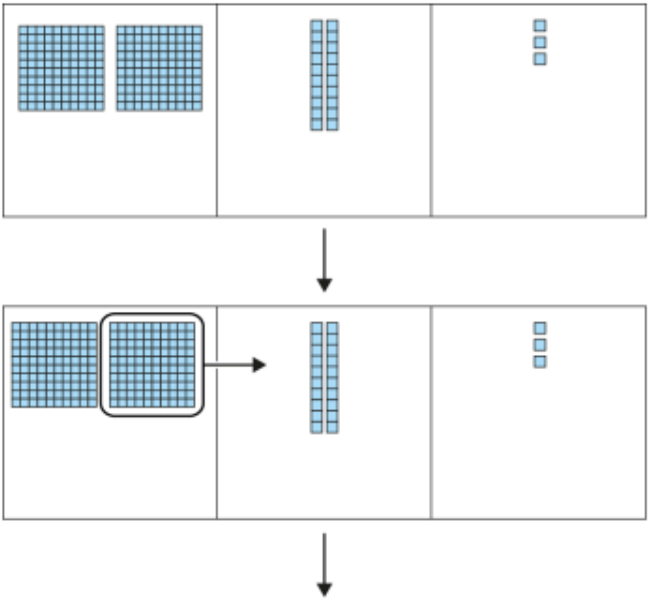
Begin by reviewing partitioning three-digit numbers into hundreds, tens and ones, and then extend to partitioning a given number in as many different ways as possible to strengthen children's understanding of place value and to facilitate the transition to double exchange, for example:

$$223 = 200 + 20 + 3$$

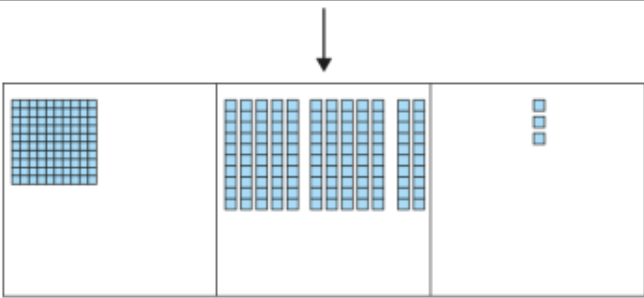
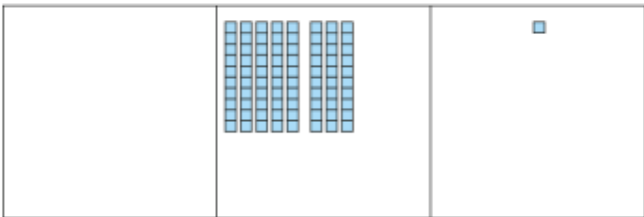
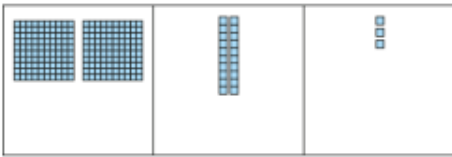
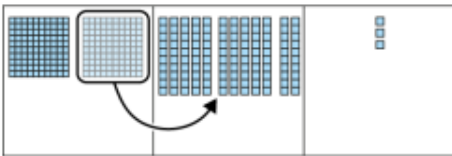
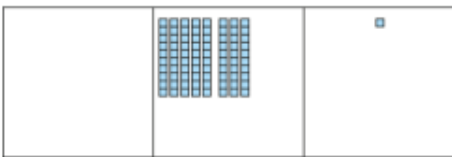
$$223 = 100 + 120 + 3$$

$$223 = 100 + 110 + 13$$



		
2:5	Now use Dienes to represent subtraction calculations that require the exchange of one hundred with ten tens. Encourage children to identify what is the same and what is different compared to exchanging one ten with ten ones.	223 - 142 

1.21 Column subtraction

		 																																				
2:6	Then, introduce the abstract representation alongside the Dienes. After demonstrating several examples, children can again work in pairs moving between the Dienes and abstract representations, before working just in the abstract (first with place-value headings for the columns, and then without).	   <table border="1" data-bbox="1206 880 1363 1036"> <thead> <tr> <th>100s</th> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>2</td> <td>3</td> </tr> <tr> <td>- 1</td> <td>4</td> <td>2</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <table border="1" data-bbox="1206 1129 1363 1284"> <thead> <tr> <th>100s</th> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>2 1</td> <td>12</td> <td>3</td> </tr> <tr> <td>- 1</td> <td>4</td> <td>2</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <table border="1" data-bbox="1206 1377 1363 1533"> <thead> <tr> <th>100s</th> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>2 1</td> <td>12</td> <td>3</td> </tr> <tr> <td>- 1</td> <td>4</td> <td>2</td> </tr> <tr> <td>0</td> <td>8</td> <td>1</td> </tr> </tbody> </table>	100s	10s	1s	2	2	3	- 1	4	2				100s	10s	1s	2 1	12	3	- 1	4	2				100s	10s	1s	2 1	12	3	- 1	4	2	0	8	1
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2 1	12	3																																				
- 1	4	2																																				
0	8	1																																				

1.21 Column subtraction

2:7

Now repeat steps 2:5 and 2:6 for calculations that require 'working through a zero' (e.g. $404 - 257$).

100s	10s	1s
4	0	4
2	5	7

100s	10s	1s
4 ³	10	4
2	5	7

100s	10s	1s
4 ³	10 ⁹	14
2	5	7

100s	10s	1s
4 ³	10 ⁹	14
1	4	7

2:8

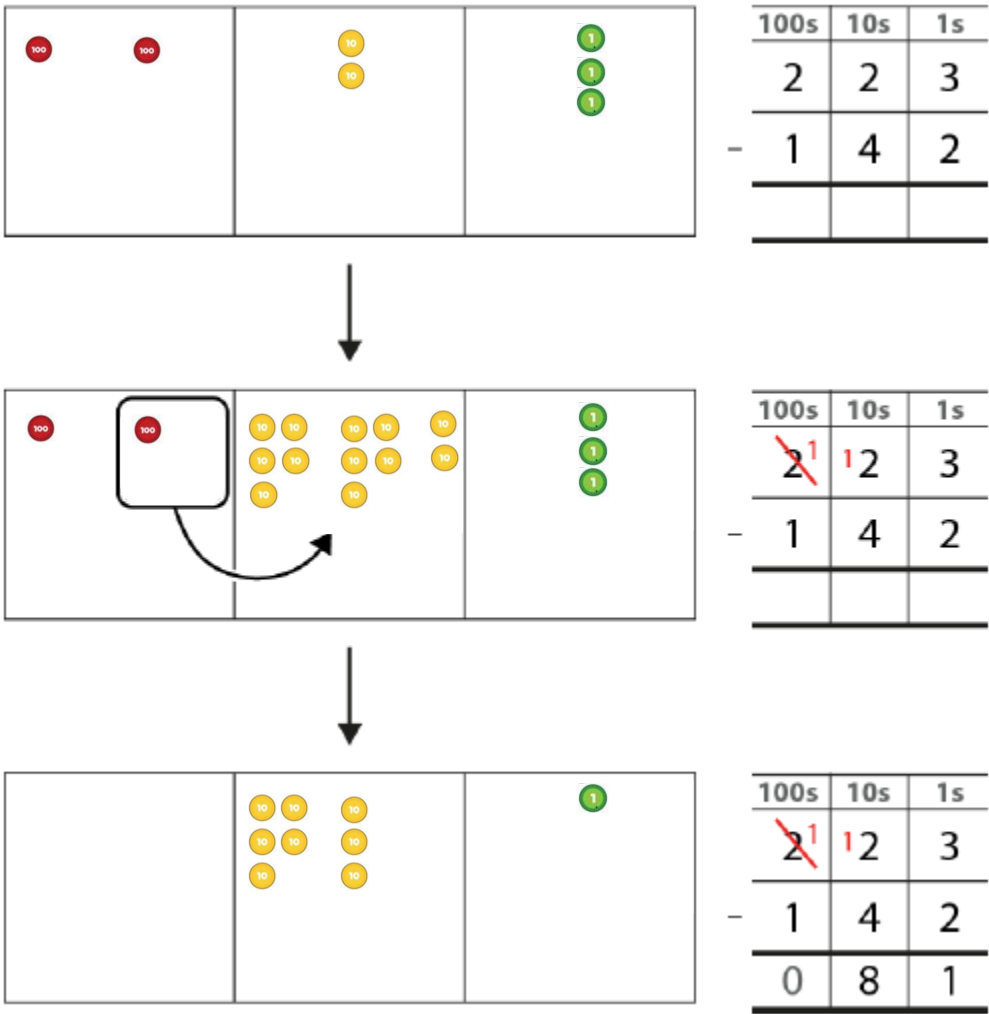
To promote depth, present children with a range of calculations and ask questions such as:

- 'Which calculations require exchange?'
- 'Which calculations require exchange only once?'
- 'Which calculations require exchange twice?'
- 'Which calculations require exchange through a zero?'

Encourage children to look at the numbers involved and justify their answers in that way, rather than actually performing the calculations.

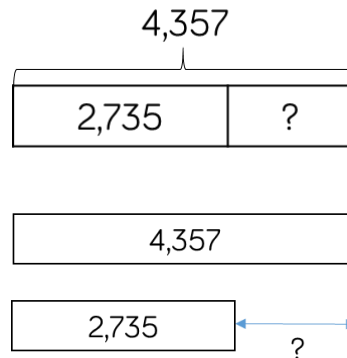
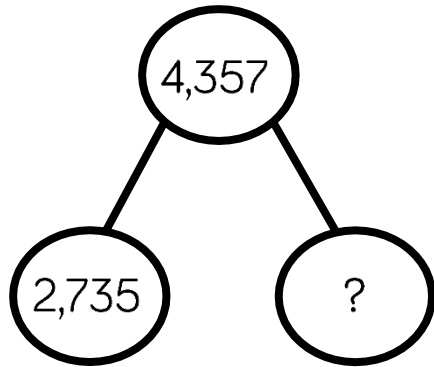
$\begin{array}{r} 563 \\ - 213 \\ \hline \end{array}$	$\begin{array}{r} 482 \\ - 197 \\ \hline \end{array}$	$\begin{array}{r} 824 \\ - 319 \\ \hline \end{array}$
$\begin{array}{r} 405 \\ - 123 \\ \hline \end{array}$	$\begin{array}{r} 316 \\ - 103 \\ \hline \end{array}$	$\begin{array}{r} 903 \\ - 124 \\ \hline \end{array}$

Column Subtraction (Place Value Counters)



Skill: Subtract numbers with up to 4 digits

Year: 4

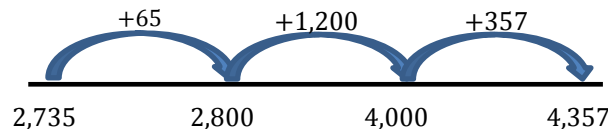


$$\begin{array}{r}
 \begin{smallmatrix} 3 & 1 \\ \cancel{4} & 3 \end{smallmatrix} 57 \\
 - 2735 \\
 \hline
 1622
 \end{array}$$

$$4,357 - 2,735 = 1,622$$

Refer to Year 3 column subtraction method and apply with decimals.

Ensure Base 10 is used before moving onto using place value counters.



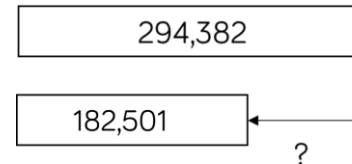
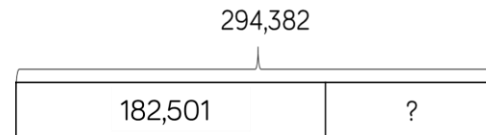
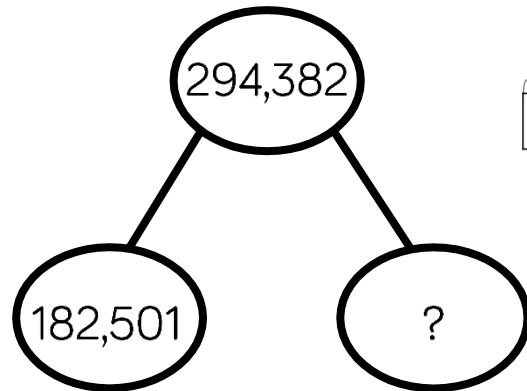
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

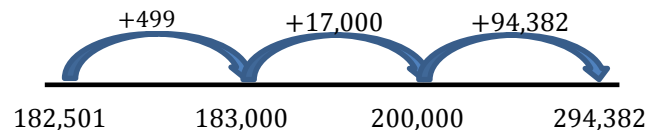
Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with more than 4 digits

Year: 5/6



$$294,382 - 182,501 = 111,881$$



Refer to Year 3 column subtraction method and apply with numbers greater than 4 digits.

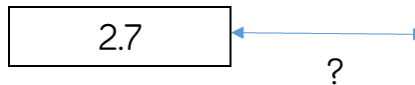
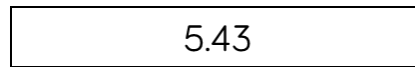
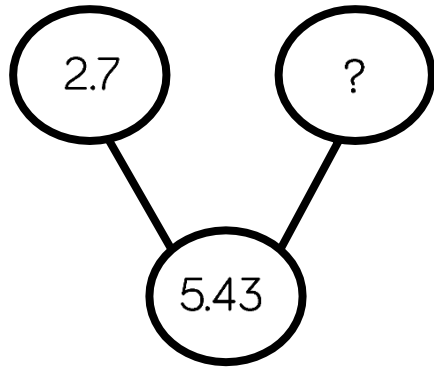
	2	9	3	¹ 3	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

Skill: Subtract with up to 3 decimal places

Year: 5

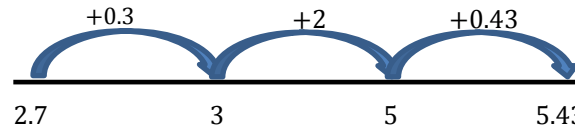


$$5.43 - 2.7 = 2.73$$

$$\begin{array}{r} \overset{4}{\cancel{5}}.\overset{1}{\cancel{4}}3 \\ - 2.7 \\ \hline 2.73 \end{array}$$

Refer to Year 3 column subtraction method and apply with decimals.

Base 10 to be used before place value counters to develop children's understanding of decimals.



Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.