ST MARY'S CHURCH OF ENGLAND PRIMARY SCHOOL

Year 1 - 6

Calculation Policy

Multiplication and Division

Notes and Guidance

St Mary’s Church of England Primary School Calculation Policy adapted from White Rose Maths, and NCETM professional development materials.

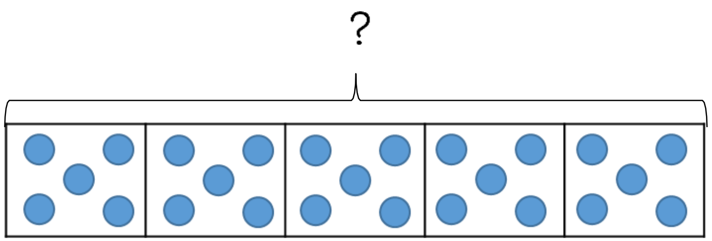
This document is broken down into addition and subtraction, and multiplication and division.

At the start of each policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

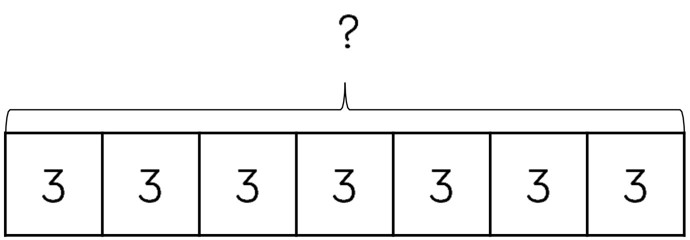
Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

There is an overview of the skills linked to year groups to support consistency throughout school. A glossary of terms is provided at the end of the calculation policy to support the understanding of the key language used to teach the four operations.

Bar Model

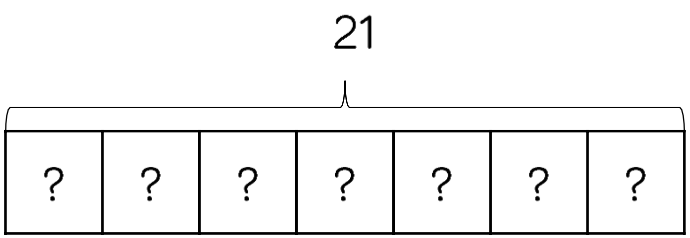


$5 \times 5 = 25$

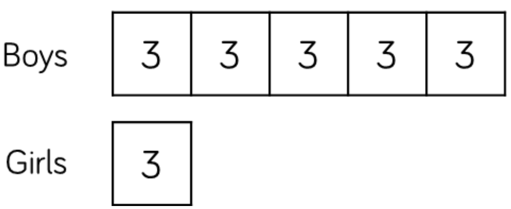


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

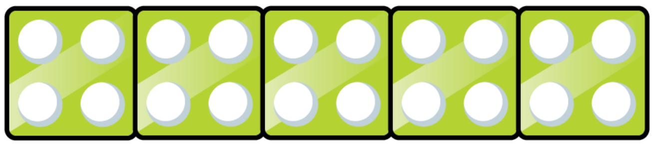
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

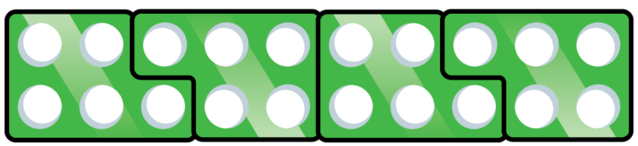
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

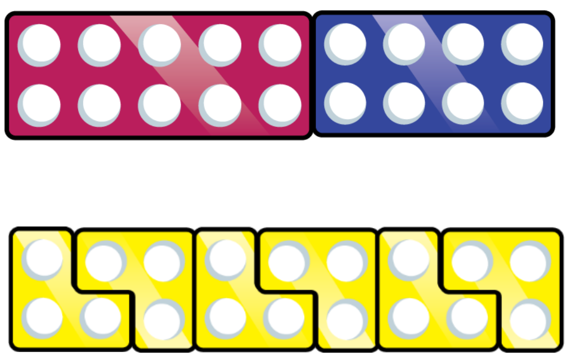
Number Shapes



$5 \times 4 = 20$
$4 \times 5 = 20$



$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$

Benefits

Number shapes support children's understanding of multiplication as repeated addition.

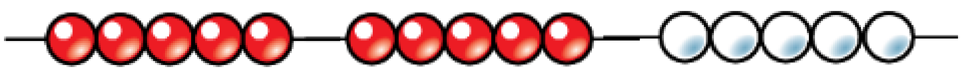
Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd $\times$ odd $=$ even, odd $\times$ even $=$ odd, even $\times$ even $=$ even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$

$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

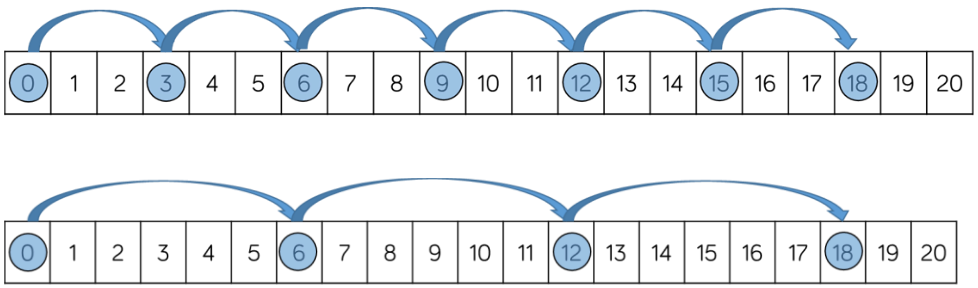
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

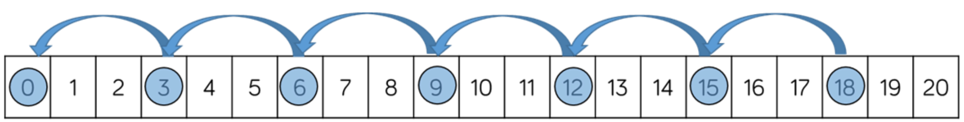
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$$6 \times 3 = 18$$
$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

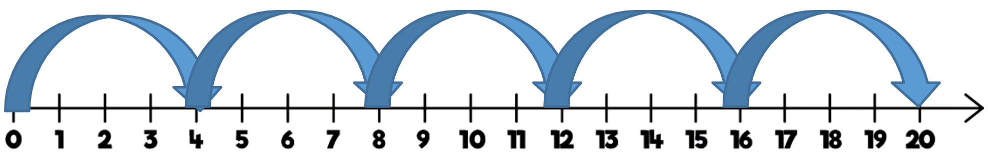
Benefits

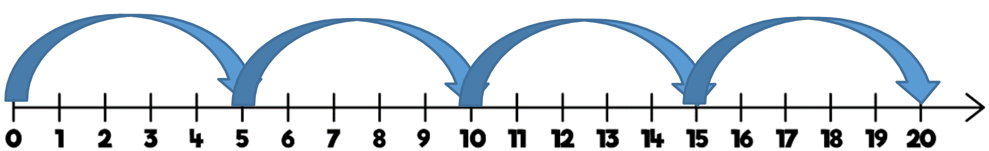
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

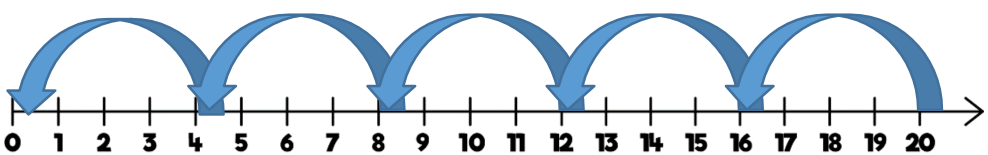
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)





$$4 \times 5 = 20$$
$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

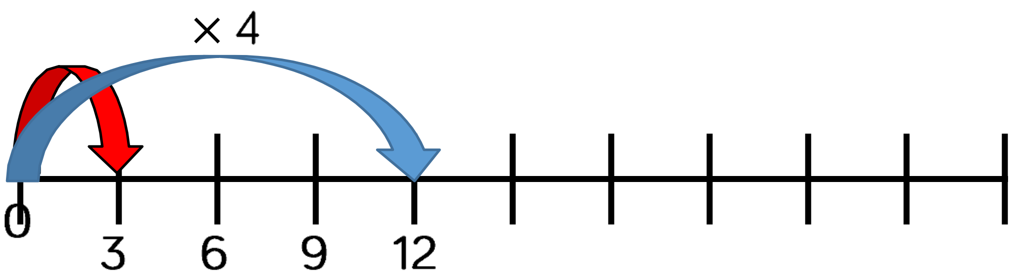
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

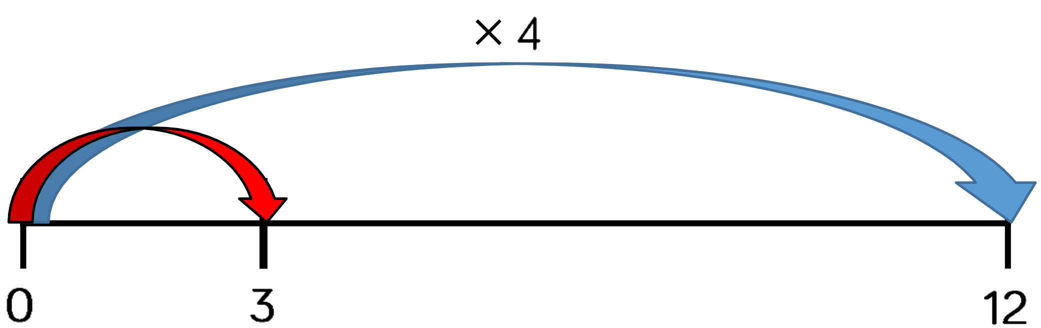
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

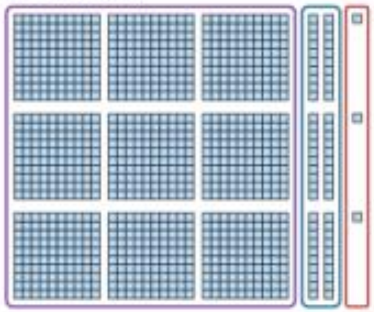
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)

321 × 3

Dienes representation:



Multiplication algorithm – expanded layout:

	100s	10s	1s	
	3	2	1	
×			3	
			3	3 × 1 ones = 3 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	3	

Multiplication algorithm – compact layout:

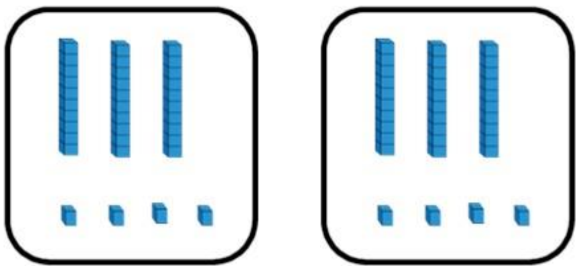
	3	2	1
×			3
	9	6	3

Benefits

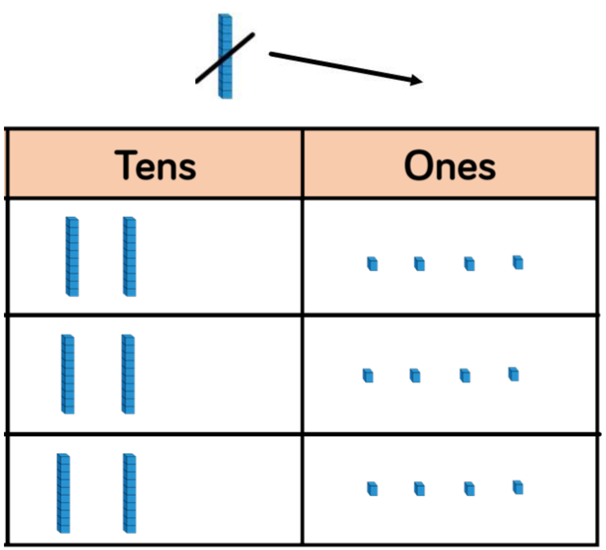
Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

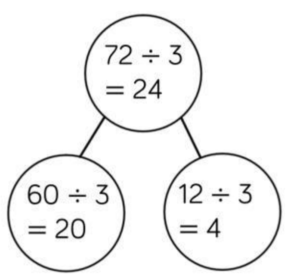
Base 10/Dienes (division)



$$68 \div 2 = 34$$



$$72 \div 3 = 24$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

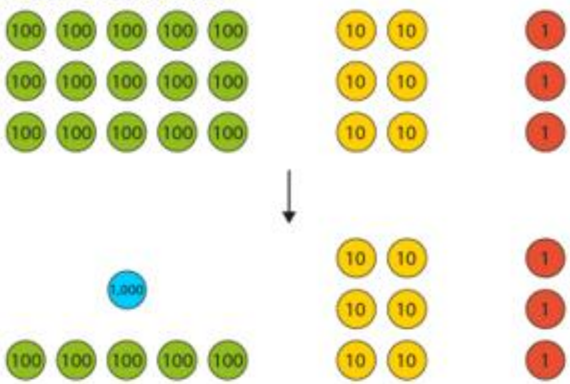
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

521 × 3

Place-value counters:



Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		5	2	1
×				3
				3
			6	0
	1	5	0	0
	1	5	6	3

3 × 1 ones = 3 ones

3 × 2 tens = 6 tens

3 × 5 hundreds = 15 hundreds

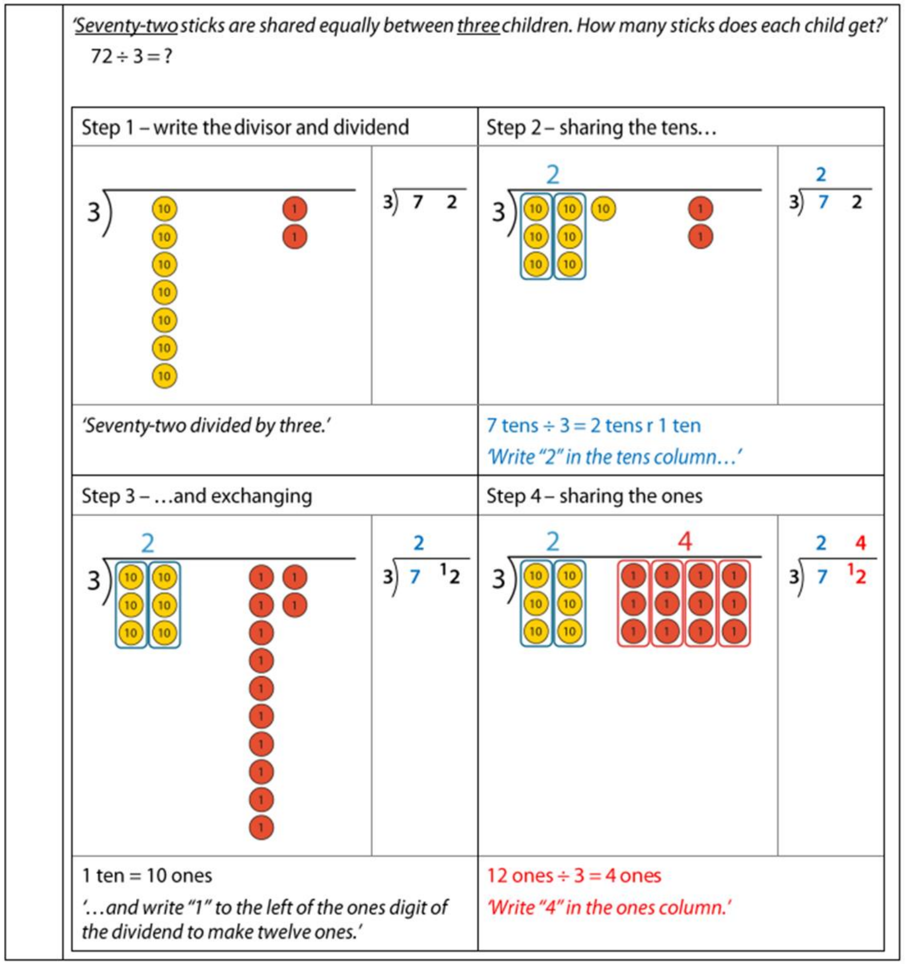
= 1 thousand + 5 hundreds

Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place Value Counters (division)



Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

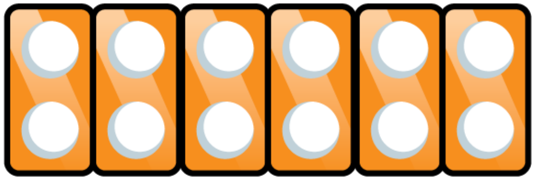
Times Tables

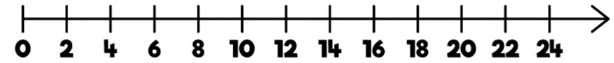
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 2-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 5-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 10-times table	2	Hundred square Number shapes Counters Money	Ten frames Bead strings Number lines Base 10

Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 3-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 4-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 8-times table	3	Hundred square Number shapes	Bead strings Number tracks Everyday objects
Recall and use multiplication and division facts for the 6-times table	4	Hundred square Number shapes	Bead strings Number tracks Everyday objects

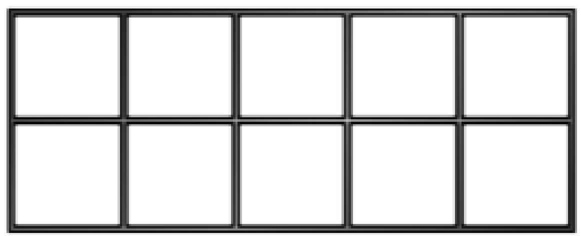
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 7-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 9-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 11-times table	4	Hundred square Base 10	Place value counters Number lines
Recall and use multiplication and division facts for the 12-times table	4	Hundred square Base 10	Place value counters Number lines

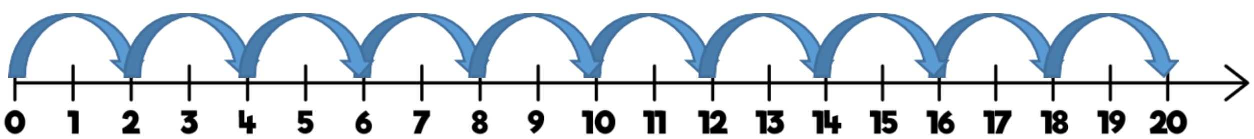
Skill: 2 times table	Year: 2





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



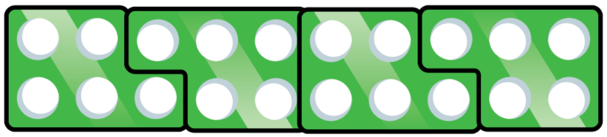


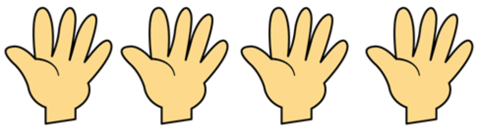
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

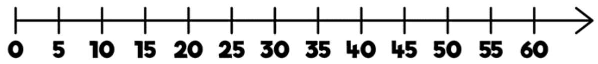
Use different models to develop fluency.

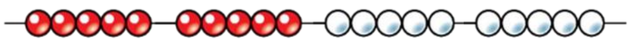
Skill: 5 times table	Year: 2

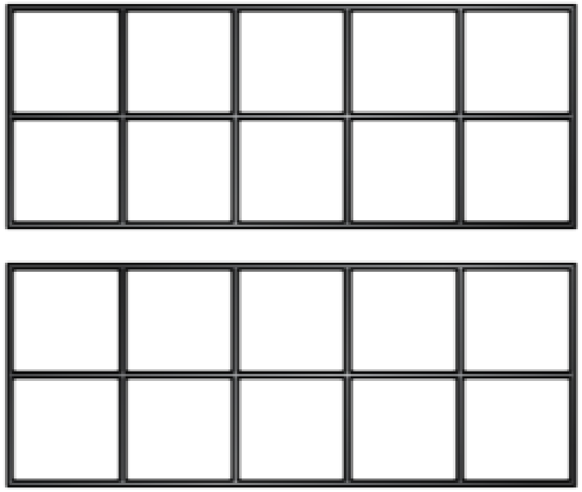


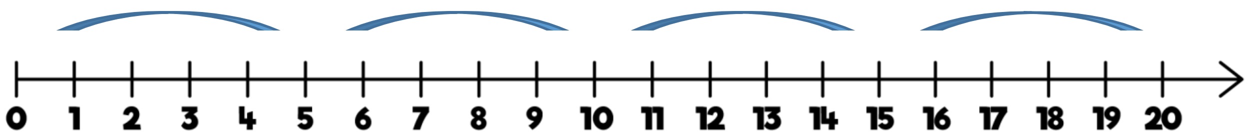


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50





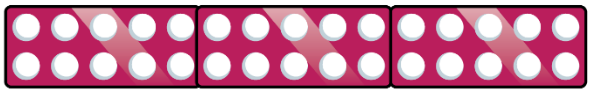


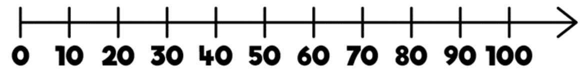


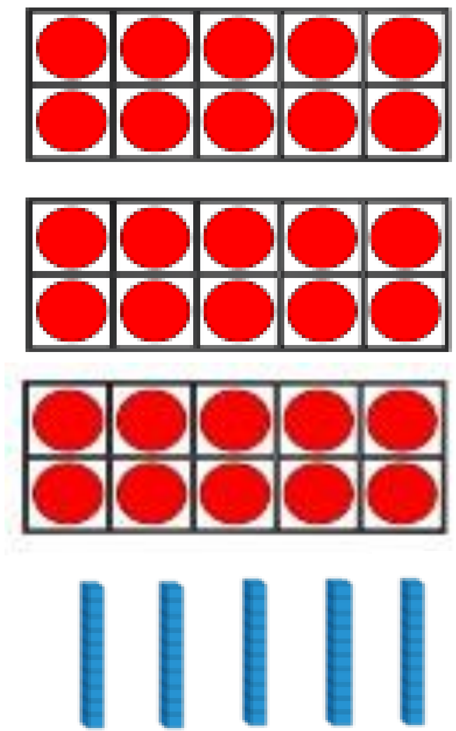
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Skill: 10 times table	Year: 2





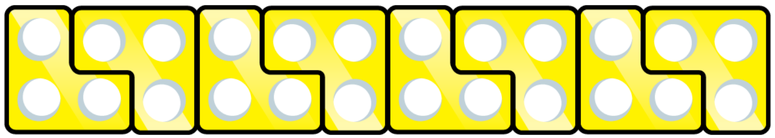


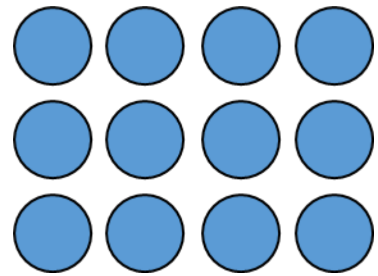
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

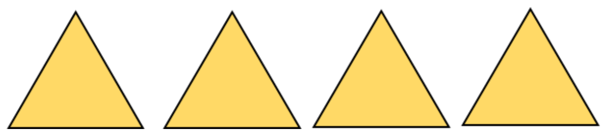
Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits- the ones are always 0, and the tens increase by 1 ten each time.

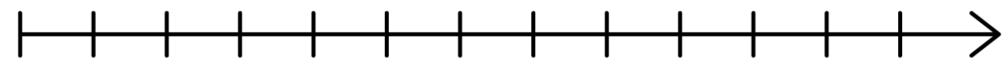
Skill: 3 times table	Year: 3





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

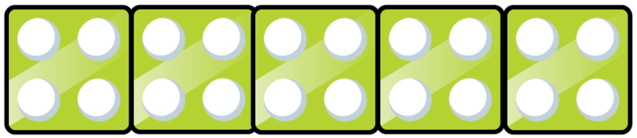




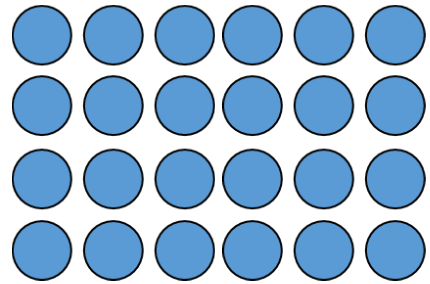
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

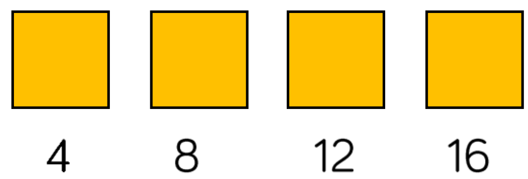
Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

Skill: 4 times table	Year: 3

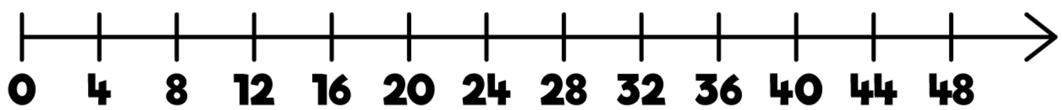


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



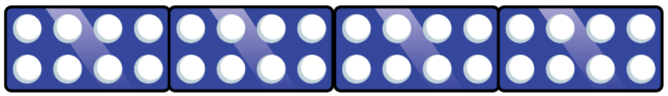


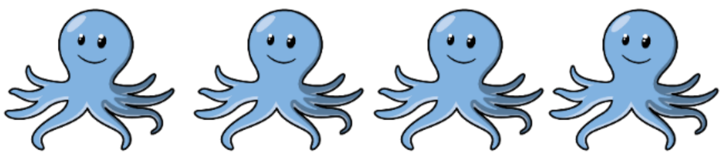
4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 8 times table	Year: 3

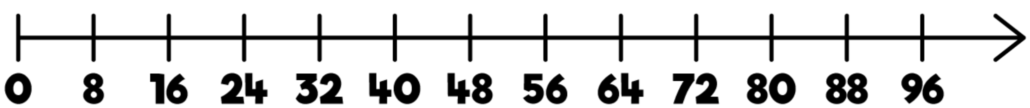




8 16 24 32

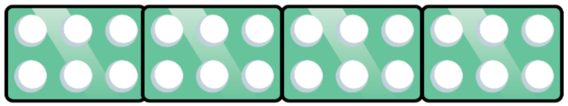
8	16	24	32	40
48	56	64	72	80

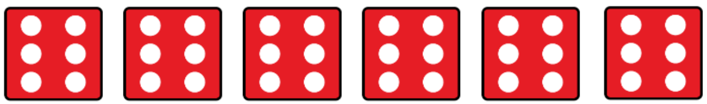
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square.
Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples.
Highlight that all the multiples are even using number shapes to support.

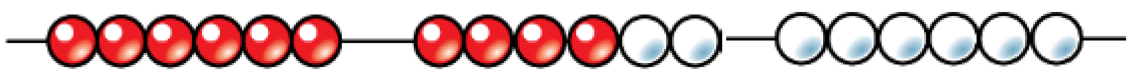
Skill: 6 times table	Year: 4

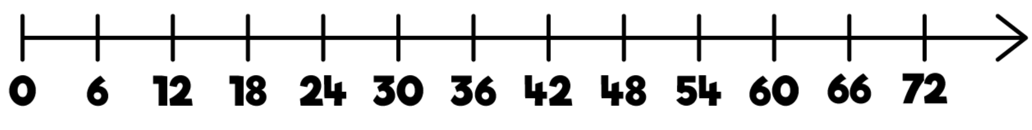




6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



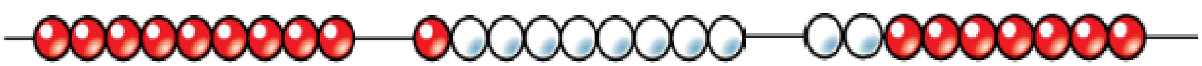


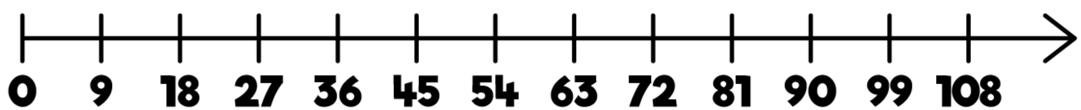
Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 9 times table	Year: 4

9	18	27	36	45
54	63	72	81	90

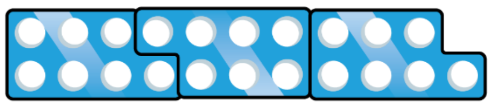
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100





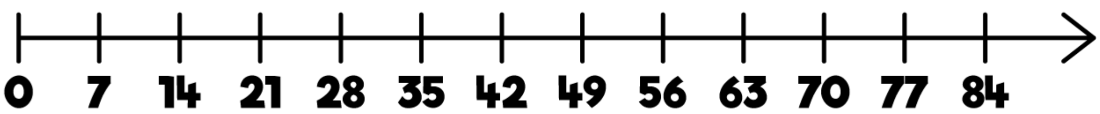
Encouraging daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

Skill: 7 times table	Year: 4



7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



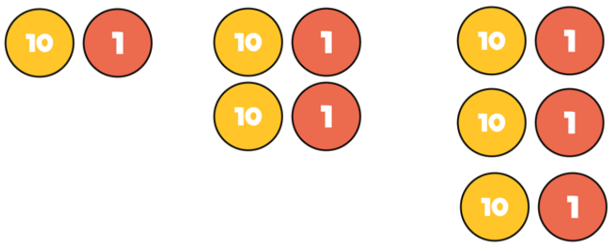
Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square.

The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity.

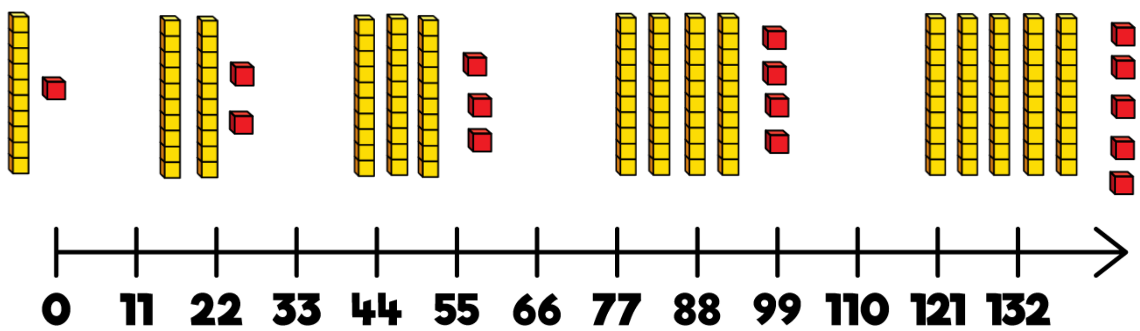
Children can still see the odd, even pattern in the multiples using number shapes to support.

Skill: 11 times table	Year: 4

11	22	33	44	55	66
77	88	99	110	121	132



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

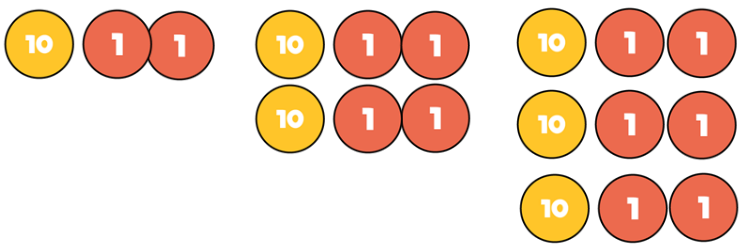


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

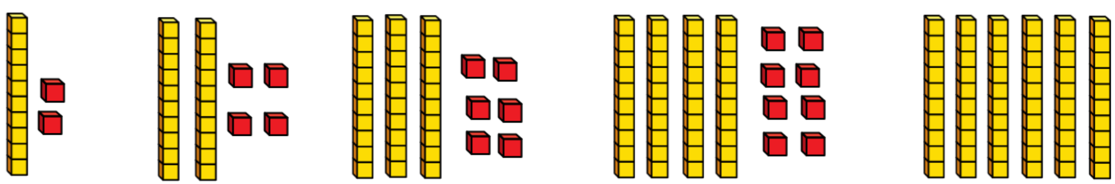
Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

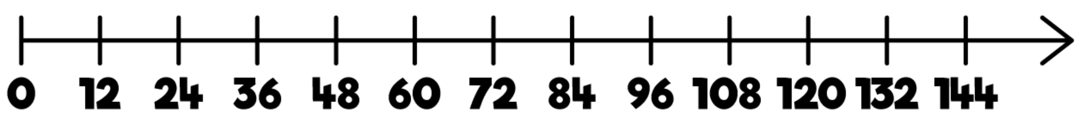
Skill: 12 times table	Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



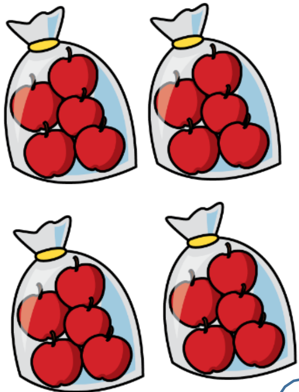
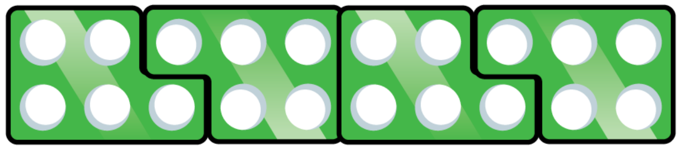
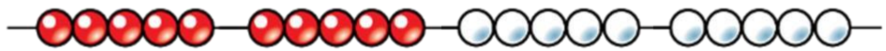
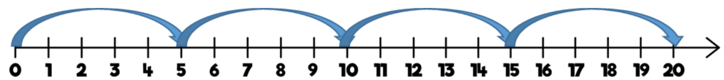
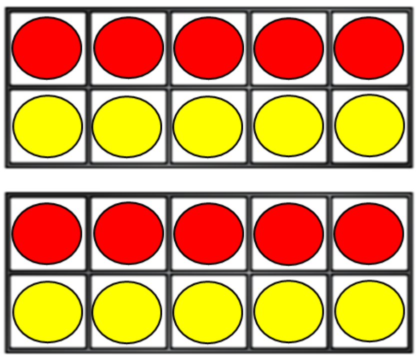
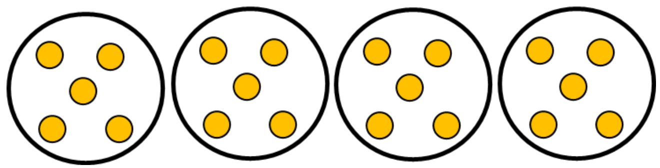
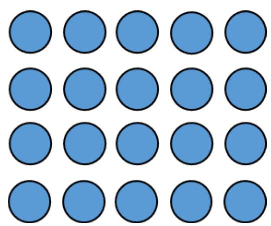


Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

Multiplication

Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Short written method Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method

Skill	Year	Representations and models	
Multiply 2-digit by 2-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Skill: Solve 1-step problems using multiplication	Year: 1/2
    One bag holds 5 apples. How many apples do 4 bags hold?    $5 + 5 + 5 + 5 =$ $4 \times 5 = 20$ $5 \times 4 = 20$	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.

Skill: Multiply 2-digit numbers by 1-digit numbers	Year: 3/4

	H	T	O		
		3	4		
×			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method.
The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

1:2

Now progress to multiplying a non-teen two-digit number by a single-digit number. Dienes can be used to represent the numbers, in a similar way to *Spine 1: Number, Addition and Subtraction*, segments *1.15*, *1.16*, *1.20* and *1.21*.

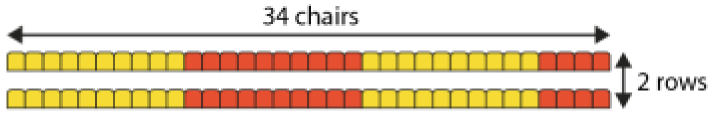
Present a problem, such as: *'There are two rows, each with thirty-four chairs. How many chairs are there altogether?'*

Note that, for now, numbers should be chosen to avoid the need to regroup.

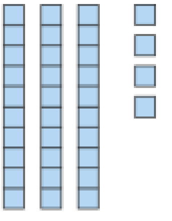
Ask children to partition the 34, describing it in terms of tens and ones, then represent it using Dienes. Then make another copy of the Dienes representation of 34, since we are multiplying by two. Finally, bring the tens together and the ones together, to represent the product. Record equations and use unitising language as you work through the steps, as shown opposite and on the next page.

Repeat the process for a range of *two-digit × single-digit* calculations, without regrouping, until children are confident. Use examples where the two-digit number is sometimes presented as the first factor and sometimes as the second factor.

'There are two rows, each with thirty-four chairs. How many chairs are there altogether?'



Step 1 – partition thirty-four:

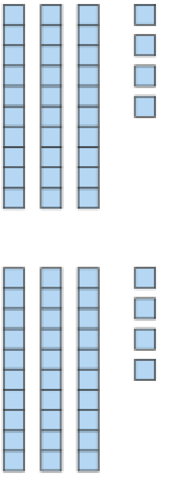


$34 = 30 + 4$

- *'Thirty-four is equal to three tens and four ones.'*

$34 = 3 \text{ tens} + 4 \text{ ones}$

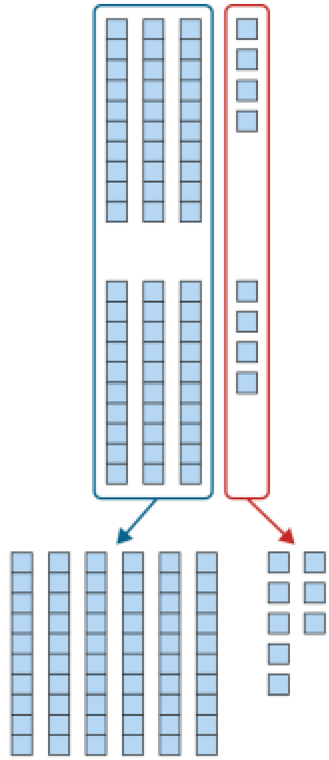
Step 2 – make a copy of the representation (multiply by two):



34×2

- *'Three-tens-and-four-ones multiplied by two.'*

Step 3 – multiply the tens and ones and recombine:



$34 \times 2 = 30 \times 2 + 4 \times 2$

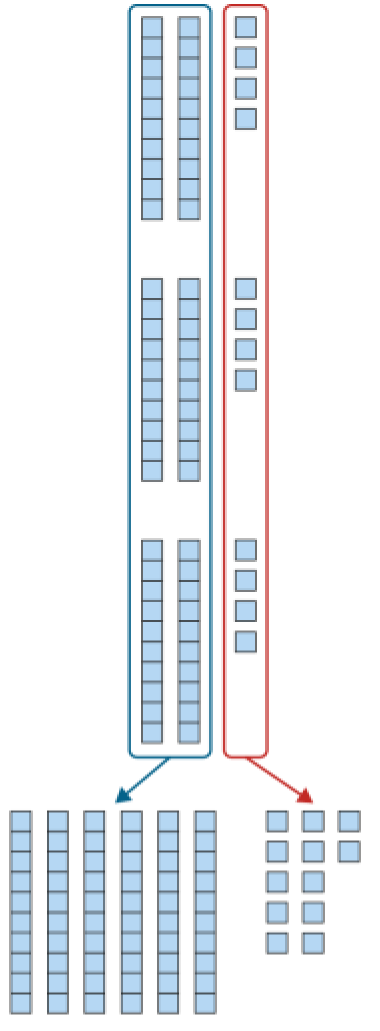
$= \quad 60 \quad + \quad 8$

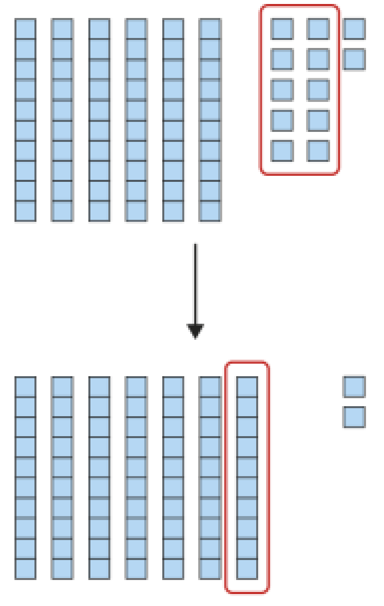
$= 68$

- *'Three-tens-and-four-ones multiplied by two is equal to three tens multiplied by two and four ones multiplied by two.'*

3 tens × 2 = 6 tens

4 ones × 2 = 8 ones

1:3	Once children are comfortable with the representations, equations and language used in step *1:2*, move to an example that requires regrouping of ones into tens, for example, *'There are three rows, each with twenty-four chairs. How many chairs are there altogether?'* Follow the same sequence as for step *1:2*, then regroup the ones (note that only the final two steps are shown opposite and on the next page). Work through several examples, working towards the generalisation: ***'If there are ten or more ones, we must regroup the ones into tens and ones.'*** Use examples where the two-digit number is sometimes presented as the first factor and sometimes as the second factor. Also, include examples for which the ones are regrouped into *more* than one ten (and some ones), but avoiding the need to regroup the tens into hundreds, for example: $28 \times 3 = 20 \times 3 + 8 \times 3$ $ = \quad 60 \quad + \quad 24$	*'There are three rows, each with twenty-four chairs. How many chairs are there altogether?'* Multiply the tens and ones and recombine:  $24 \times 3 = 20 \times 3 + 4 \times 3$ $ = \quad 60 \quad + \quad 12$ • *'Two-tens-and-four-ones multiplied by three is equal to two tens multiplied by three and four ones multiplied by three.'* 2 tens × 3 = 6 tens 4 ones × 3 = 12 ones

		Regroup the ones into tens and ones:  $24 \times 3 = 60 + 10 + 2$ $= 70 + 2$ $= 72$ • *'Twelve ones is equal to one ten and two ones.'* 12 ones = 1 ten + 2 ones

Repeat using place value counters where applicable.

Skill: Multiply 3-digit numbers by 1-digit numbers	Year: 3/4

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

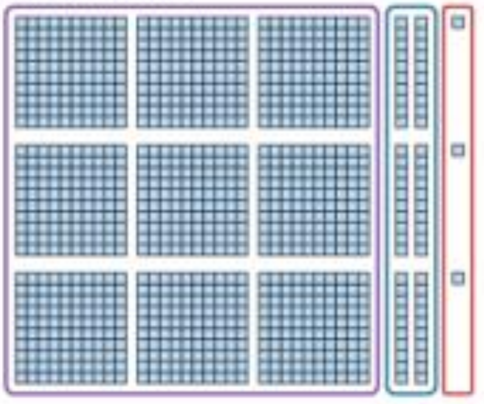
When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.

Base 10 and place value counters continue to support the understanding of the written method.

Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

321×3

Dienes representation:



Multiplication algorithm – expanded layout:

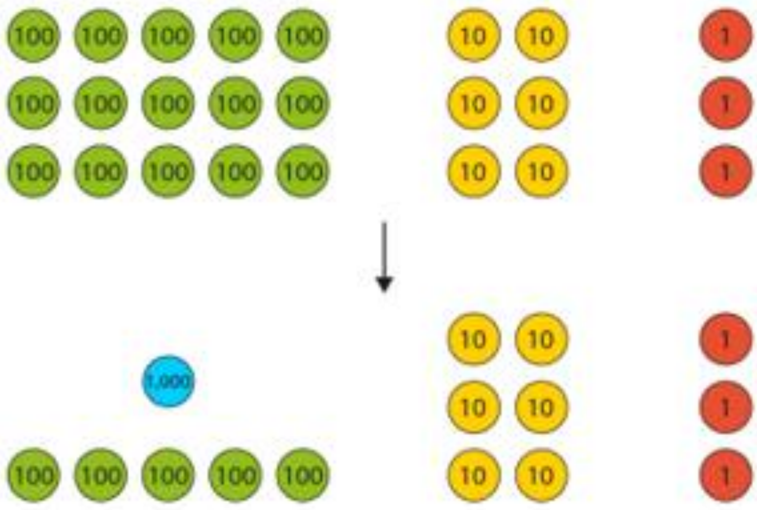
	100s	10s	1s	
	3	2	1	
×			3	
			3	3×1 ones = 3 ones
		6	0	3×2 tens = 6 tens
	9	0	0	3×3 hundreds = 9 hundreds
	9	6	3	

Multiplication algorithm – compact layout:

	3	2	1
×			3
	9	6	3

521×3

Place-value counters:



Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		5	2	1
×				3
				3
			6	0
	1	5	0	0
	1	5	6	3

3×1 ones $= 3$ ones

3×2 tens $= 6$ tens

3×5 hundreds $= 15$ hundreds

$= 1$ thousand $+ 5$ hundreds

Skill: Multiply 4-digit numbers by 1-digit numbers	Year: 5

	Th	H	T	O
	1	8	2	6
x				3
	5	4	7	8
	2		1	

Refer to Year 4 multiplication method and apply with 4 digit by 1 digit multiplication.

Use only place value counters due to the size of numbers involved.

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers	Year: 5
<table><tr><td>×</td><td>20</td><td>2</td></tr><tr><td>30</td><td>600</td><td>60</td></tr><tr><td>1</td><td>20</td><td>2</td></tr></table> <table><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td></td><td>2</td><td>2</td></tr><tr><td>×</td><td></td><td>3</td><td>1</td></tr><tr><td></td><td></td><td>2</td><td>2</td></tr><tr><td></td><td>6</td><td>6</td><td>0</td></tr><tr><td></td><td>6</td><td>8</td><td>2</td></tr></table>	When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

4:2

Now spend some time working through the long multiplication algorithm using an expanded layout as shown below and on the next page. Work through one step at a time in a similar way to how the short multiplication algorithm was introduced in segment *2.14 Multiplication: partitioning leading to short multiplication*. Remember that, at this stage, we are focusing on multiplications that only require *regrouping of tens into hundreds*.

Although the two two-digit factors can be written in either order, it is beneficial to get children into the practice of writing the largest factor first, as this will become necessary when one of the factors has more than two digits (see steps *4:9* and *4:11*).

Draw attention to the following key points, encouraging children to repeat the instructions at each step:

- *'First, write the larger factor: 31.'*
- *'Then write the smaller factor below, lining up the digits: 24.'*
- *'Now multiply by the ones digit to give a partial product: to multiply thirty-one by four…'*
- *'…multiply one one by four to give four ones; write "4" in the ones column…'*
- *'…then multiply three tens by four to give twelve tens…'*
- *'…and regroup: twelve tens is equal to one hundred, and two tens; write "1" in the hundreds column and "2" in the tens column.'*
- *'Then, place a zero to show that it's ten times the size.'*
- *'Next, multiply by the tens digit to give a partial product: to multiply thirty-one by twenty…'*
- *'…multiply one one by two tens to give two tens; write "2" in the tens column…'*
- *'…then multiply three tens by two tens to give six hundreds; write "6" in the hundreds column.'*
- *'Finally, add the partial products: 124 plus 620 is equal to 744.'*

If children need additional support with regrouping, they can use place-value counters as in segment *2.14*.

Look more closely at the expanded algorithm and point out that, just like in short multiplication:

- when the factors are recorded, the digits are aligned correctly (ones with ones, etc.)
- when the partial products are recorded, the digits are aligned correctly.

Complete a few more examples (e.g. *32 × 14* and *34 × 22*) together in the same way, talking through each step.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	100s	10s	1s
		3	1
×		2	4

Step 2 – multiply the ones digit by the ones digit

	100s	10s	1s	
		3	1	
×		2	4	
			4	4 × 1 one = 4 ones

Step 3 – multiply the tens digit by the ones digit and regroup

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	4 × 3 tens = 12 tens
				= 1 hundred + 2 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		3	1
×		2	4
	1	2	4
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
		2	0	2 tens × 1 one = 2 tens

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
	6	2	0	2 tens × 3 tens = 6 hundreds

Step 7 – add the partial products

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

4:3

When children are confident using the expanded layout from step *4:2*, compare it with the compact layout they were shown in step *4:1* and ask:

- *'What's the same?'*
- *'What's different?'*

- *'What's the same?'*
- *'What's different?'*

Multiplication algorithm – expanded layout:

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Multiplication algorithm – compact layout:

		3	1
×		2	4
	1	2	4
	6	2	0
	7	4	4

Skill: Multiply 3-digit numbers by 2-digit numbers	Year: 5

Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_{1}7$	$_{1}0$	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

4:9 Once children are confident multiplying any two-digit number by any other two-digit number, build on this by multiplying three-digit numbers by two-digit numbers (e.g. *28 × 312*).

Follow the same process as earlier (for example, step *4:7*), asking children to describe the instructions at each step in laying out and applying the algorithm. You may find that, by now, children begin to shorten the descriptive language they use to reason through the algorithm as they become more confident with the strategy. Point out that we always write the three-digit number first and then the two-digit number.

Begin with the expanded layout until children are confident moving to the compact layout without place-value headings. You may wish to compare the expanded layout with the compact layout for the first few examples.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	1,000s	100s	10s	1s
		3	1	2
×			2	8

Step 2 – multiply the ones digit by the ones digit and regroup

			1	
	1,000s	100s	10s	1s
		3	1	2
×			2	8
				6

8 × 2 ones = 16 ones
= 1 ten + 6 ones

Step 3 – multiply the tens digit by the ones digit and add the regrouped tens

			~~1~~	
	1,000s	100s	10s	1s
		3	1	2
×			2	8
			9	6

8 × 1 ten = 8 tens
8 tens + 1 ten = 9 tens

Step 4 – multiply the hundreds digit by the ones digit and regroup

			~~1~~	
	1,000s	100s	10s	1s
		3	1	2
×			2	8
	2	4	9	6

8 × 3 hundreds = 24 hundreds
= 2 thousands + 4 hundreds

Step 5 – place a zero to show that it's ten times the size

			~~1~~	
	1,000s	100s	10s	1s
		3	1	2
×			2	8
	2	4	9	6
				0

Step 6 – multiply the ones digit by the tens digit

			$\cancel{1}$		
	1,000s	100s	10s	1s	
		3	1	2	
×			2	8	
	2	4	9	6	
			4	0	2 tens × 2 ones = 4 tens

Step 7 – multiply the tens digit by the tens digit

			$\cancel{1}$		
	1,000s	100s	10s	1s	
		3	1	2	
×			2	8	
	2	4	9	6	
		2	4	0	2 tens × 1 ten = 2 hundreds

Step 8 – multiply the hundreds digit by the tens digit

			$\cancel{1}$		
	1,000s	100s	10s	1s	
		3	1	2	
×			2	8	
	2	4	9	6	
	6	2	4	0	2 tens × 3 hundreds = 6 thousands

Step 9 – add the partial products

			$\cancel{1}$		
	1,000s	100s	10s	1s	
		3	1	2	
×			2	8	
	2	4	9	6	312 × 8
	6	2	4	0	312 × 20
	8	7	3	6	
		1			

Multiplication algorithm – compact layout:

			$\cancel{1}$	
		3	1	2
×			2	8
	2	4	9	6
	6	2	4	0
	8	7	3	6
		1		

Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 2	1 5	9 3	1 7	2
5 1	4	7 1	8	0
7	6	6	9	2
		1		

When multiplying 4-digits by 2-digits, children should be confident in the written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

4:12 Multiplication algorithm – expanded layout:

Step 1 – write the factors

	10,000s	1,000s	100s	10s	1s
		3	1	2	6
×				2	8

Step 2 – multiply the four-digit number by the ones digit, regrouping where necessary

		~~1~~	~~2~~	~~4~~		
	10,000s	1,000s	100s	10s	1s	
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8

Step 3 – place a zero to show that it's ten times the size

		~~1~~	~~2~~	~~4~~		
	10,000s	1,000s	100s	10s	1s	
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
					0	

Step 4 – multiply the four-digit number by the tens digit, regrouping where necessary

			~~1~~			
		~~1~~	~~2~~	~~4~~		
	10,000s	1,000s	100s	10s	1s	
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
	6	2	5	2	0	3,126 × 20

Step 5 – add the partial products

			~~1~~			
		~~1~~	~~2~~	~~4~~		
	10,000s	1,000s	100s	10s	1s	
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
	6	2	5	2	0	3,126 × 20
	8	7	5	2	8	

Multiplication algorithm – compact layout:

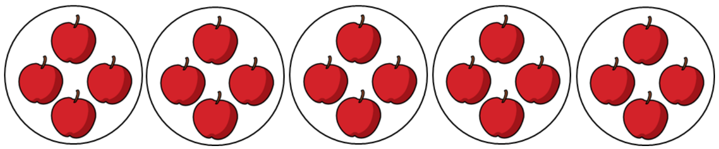
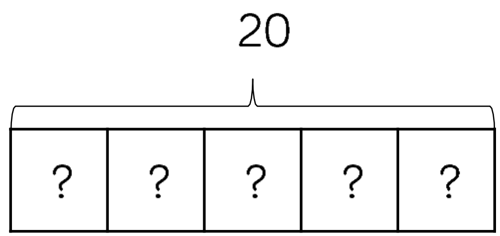
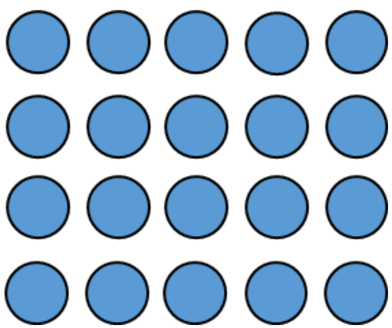
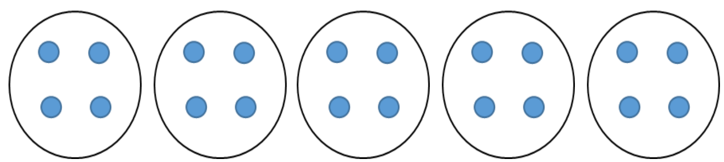
			~~1~~		
		~~1~~	~~2~~	~~4~~	
		3	1	2	6
×				2	8
	2	5	0	0	8
	6	2	5	2	0
	8	7	5	2	8

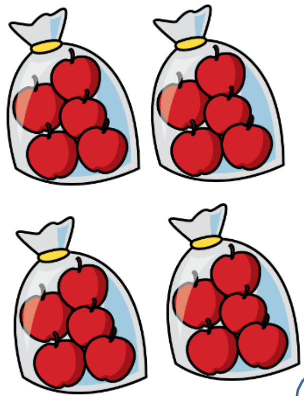
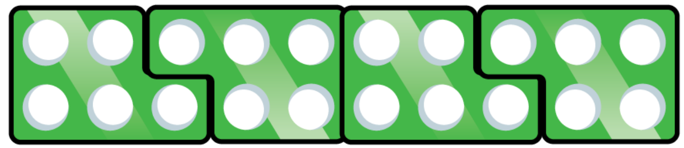
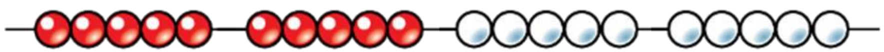
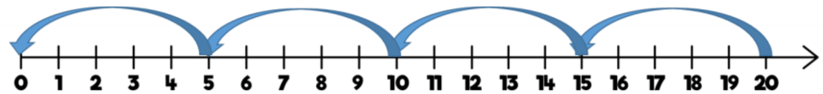
Division

Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model

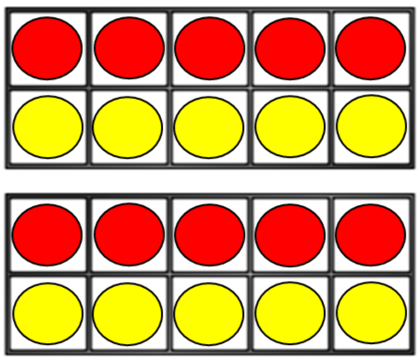
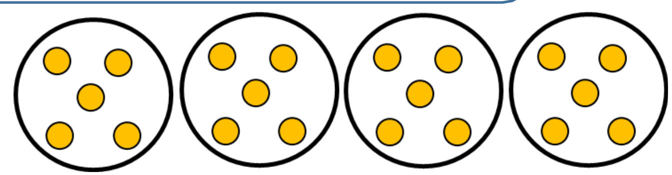
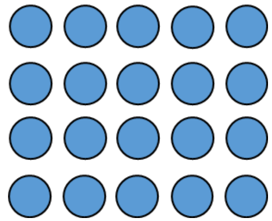
Skill	Year	Representations and models	
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Skill	Year	Representations and models	
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2
  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?   $20 \div 5 = 4$	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.

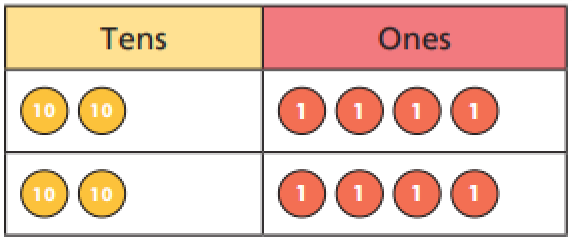
<table>
<tr><th>Skill: Solve 1-step problems using division (grouping)</th><th>Year: 1/2</th></tr>
<tr><td>





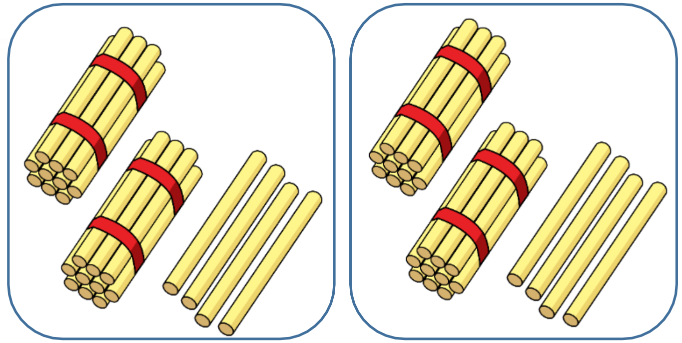
There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

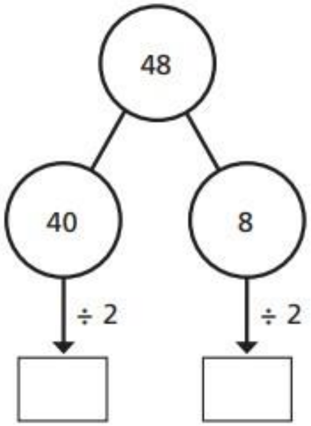




$20 \div 5 = 4$
</td><td>Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.</td></tr>
</table>

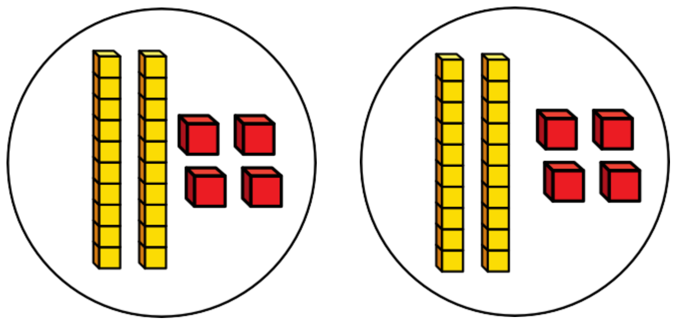
Skill: Divide 2-digits by 1-digit (sharing with no exchange)	Year: 1/2







$$48 \div 2 = 24$$

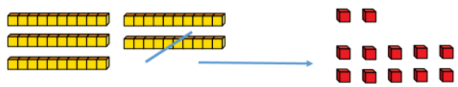


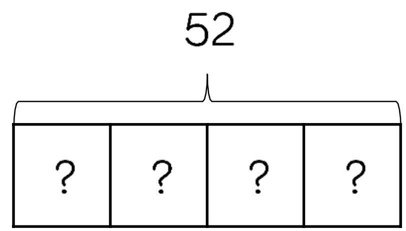
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

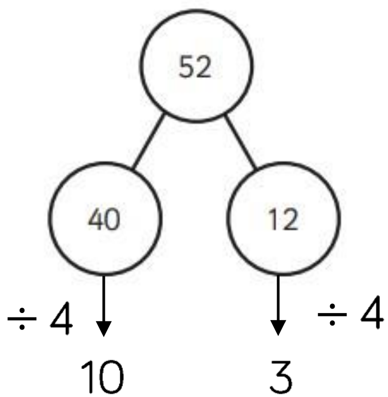
Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with exchange)	Year: 3/4





$$52 \div 4 = 13$$



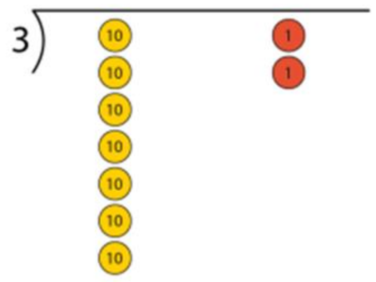
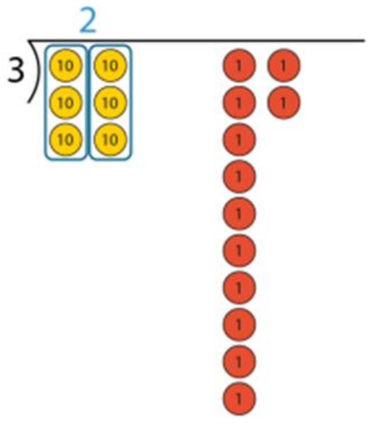
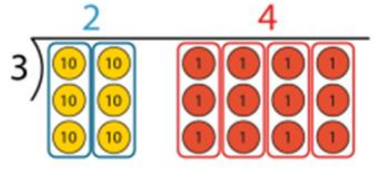
$10 + 3 = 13$

When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

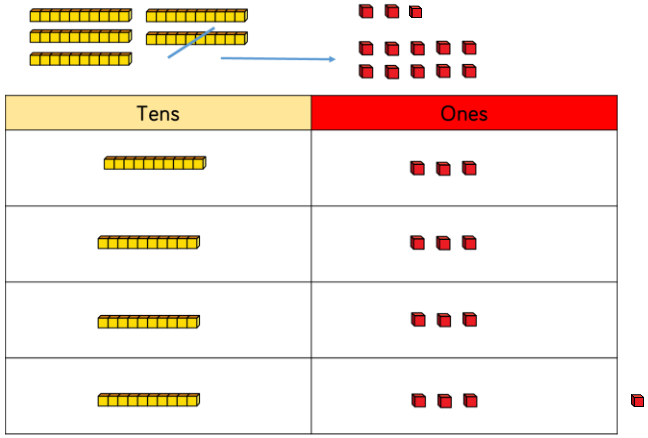
Flexible partitioning in a part-whole model supports this method.

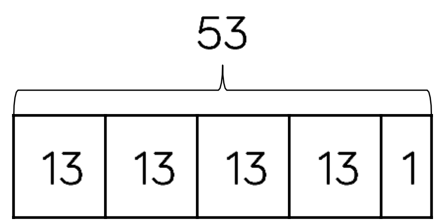
'<u>Seventy-two</u> sticks are shared equally between <u>three</u> children. How many sticks does each child get?'

$72 \div 3 = ?$

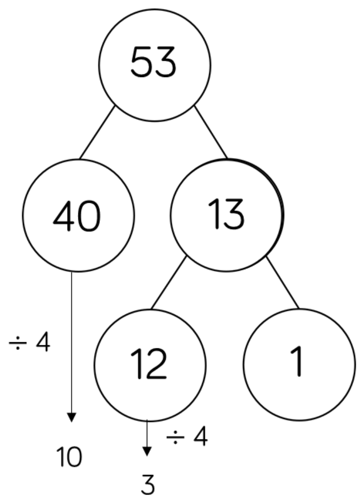
Step 1 – write the divisor and dividend		Step 2 – sharing the tens…	
	$3\overline{)\,7\quad 2}$		$\begin{array}{l}\ \ \ 2 \\ 3\overline{)\,7\quad 2}\end{array}$
'Seventy-two divided by three.'		7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*	
Step 3 – …and exchanging		**Step 4 – sharing the ones**	
	$\begin{array}{l}\ \ \ 2 \\ 3\overline{)\,7\quad {}^{1}2}\end{array}$		$\begin{array}{l}\ \ \ 2\quad 4 \\ 3\overline{)\,7\quad {}^{1}2}\end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*		12 ones ÷ 3 = 4 ones *'Write "4" in the ones column.'*	

Skill: Divide 2-digits by 1-digit (sharing with remainders)	Year: 3/4





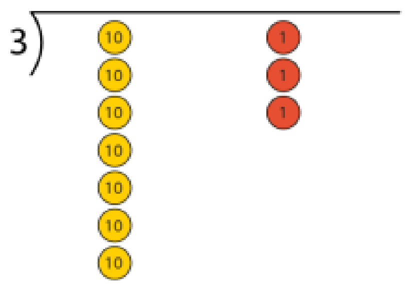
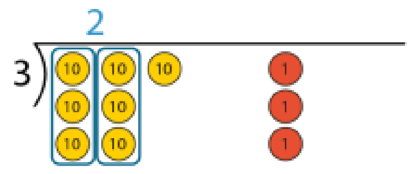
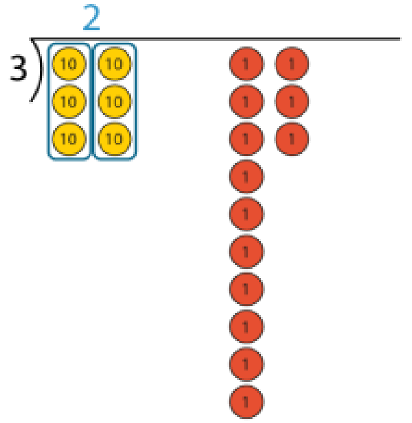
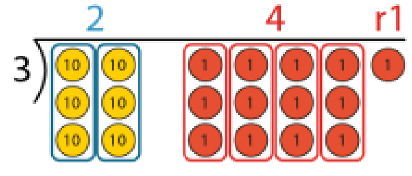
$$53 \div 4 = 13 \text{ r1}$$



When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.
Flexible partitioning in a part-whole model supports this method.

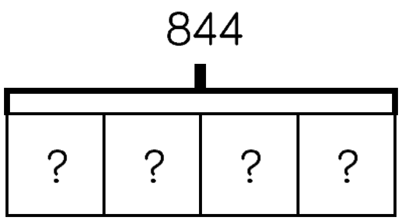
'Seventy-three sticks are shared equally between three children. How many sticks does each child get?'

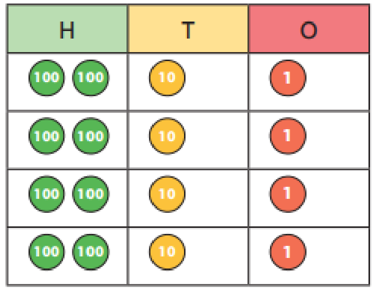
$73 \div 3 = ?$

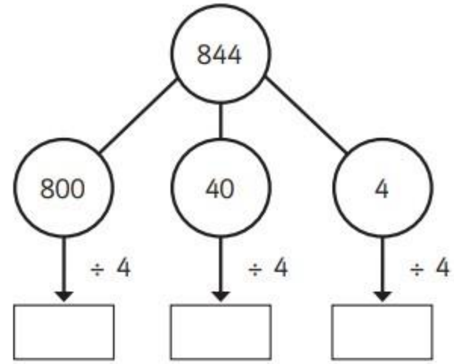
Step 1 – write the divisor and dividend		Step 2 – sharing the tens…	
	$3\overline{)7\;\;3}$		$\begin{array}{l} \;\;2 \\ 3\overline{)7\;\;3} \end{array}$
'Seventy-three divided by three.'		7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*	
Step 3 –…and exchanging		Step 4 – sharing the ones	
	$\begin{array}{l} \;\;2 \\ 3\overline{)7\;{}^{1}3} \end{array}$		$\begin{array}{l} \;\;2\;\;4\,r\,1 \\ 3\overline{)7\;{}^{1}3} \end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make thirteen ones.'*		13 ones ÷ 3 = 4 ones r 1 one *'Write "4 r 1" in the ones column.'*	

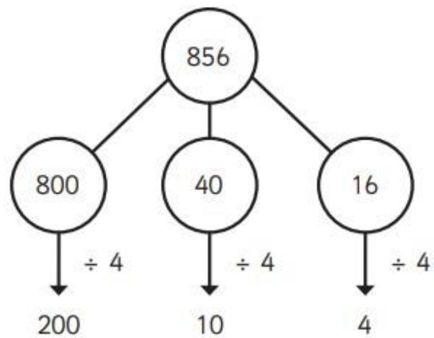
Skill: Divide 3-digits by 1-digit (sharing)	Year: 4

$844 \div 4 = 211$





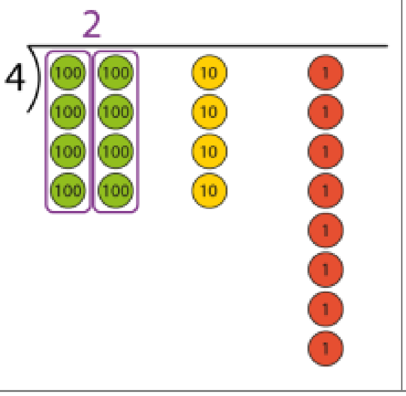


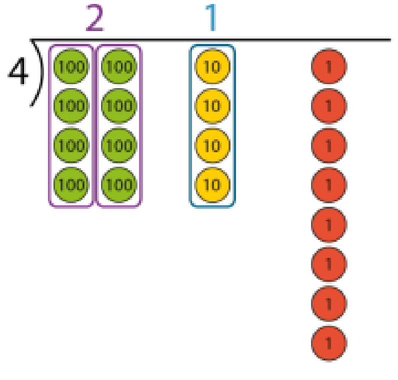
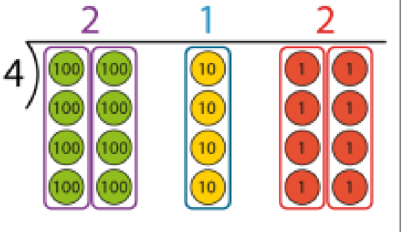


Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

'Eight hundred and forty-eight pencils are shared equally between four year groups. How many pencils does each year group get?'

$848 \div 4 = ?$

Step 1 – write the divisor and dividend		Step 2 – sharing the hundreds	
	$4\overline{)\,8\;4\;8}$		$\begin{array}{r} 2 \\ 4\overline{)\,8\;4\;8} \end{array}$
'Eight-hundred and forty-eight divided by four.'		8 hundreds ÷ 2 = 2 hundreds *'Write "2" in the hundreds column.'*	

Step 3 – sharing the tens		Step 4 – sharing the ones	
	$\begin{array}{r} 2\;1 \\ 4\overline{)\,8\;4\;8} \end{array}$		$\begin{array}{r} 2\;1\;2 \\ 4\overline{)\,8\;4\;8} \end{array}$
4 tens ÷ 4 = 1 ten *'Write "1" in the tens column.'*		8 ones ÷ 4 = 2 ones *'Write "2" in the ones column.'*	

4:3

Now move on to the calculation from step *3:2*, which requires exchange of hundreds for tens, but with no exchange of tens for ones and no overall remainder: *'Seven hundred and five exercise books are shared equally between five year groups. How many books does each year group get?'*

Work in the same way as that described in step *4:1*, but now draw particular attention to the exchange:

- *'First write the divisor: "5".'*
- *'Then draw the frame.'*
- *'Then write the dividend: "705".'*
- *'Now divide, starting with the hundreds: seven hundreds divided by five is equal to one hundred, with a remainder of two hundreds; write "1" in the hundreds column…'*
- *'and exchange the remainder: two hundreds is equal to twenty tens; write "2" to the left of the tens digit of the dividend to make twenty tens.'*
- *'Then move to the tens: twenty tens divided by five is equal to four tens; write "4" in the tens column.'*
- *'Then move to the ones: five ones divided by five is equal to one one; write "1" in the ones column.'*

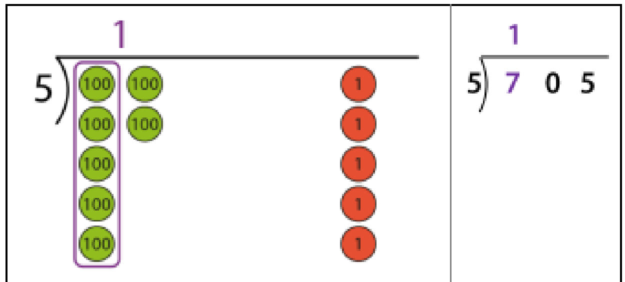
Note that only the middle steps – division of the hundreds and exchange – are fully illustrated opposite, but place-value counters should be used throughout, alongside the written algorithm, for the first example with exchange.

Work through a variety of similar calculations (e.g. *786 ÷ 3*, *640 ÷ 4* and *576 ÷ 3*), gradually removing the

'Seven hundred and five exercise books are shared equally between five year groups. How many books does each year group get?'

$705 \div 5 = ?$

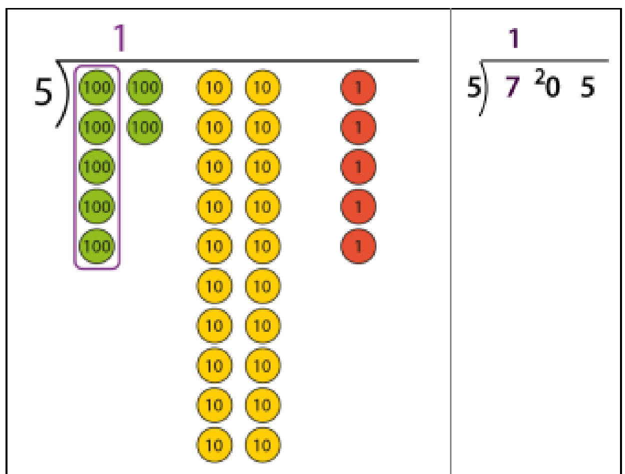
Sharing the hundreds:



7 hundreds ÷ 5 = 1 hundred r 2 hundreds

'Write "1" in the hundreds column…'

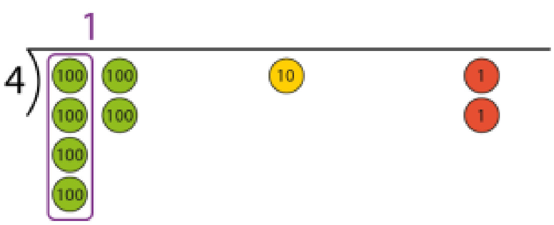
Exchanging:

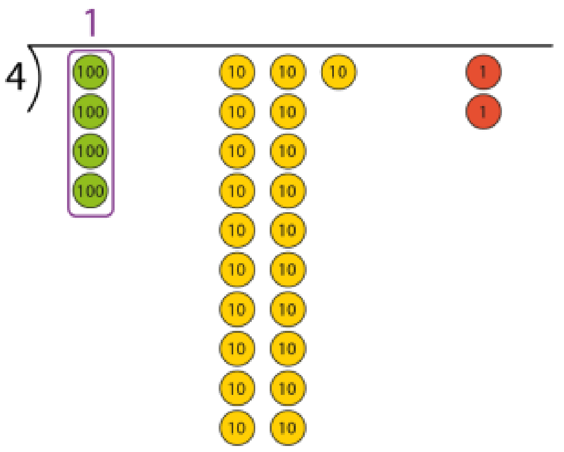
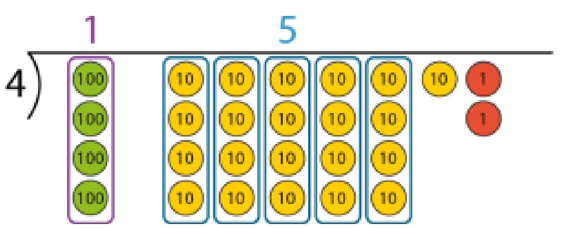
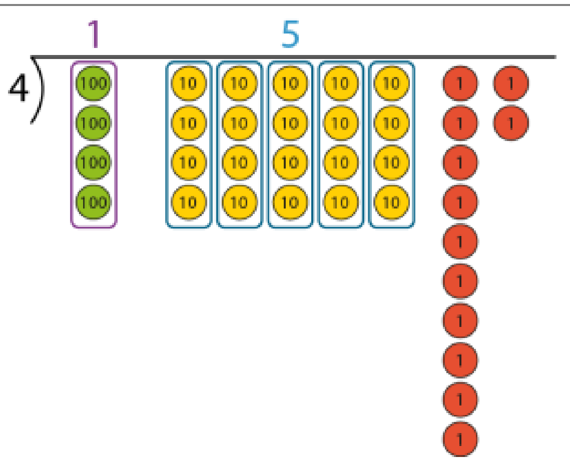


2 hundreds = 20 tens

'…and write "2" to the left of the tens digit of the dividend to make twenty tens.'

4:5	Repeat steps *4:3* and *4:4* for a range of calculations including examples involving: • no exchange but an overall remainder (e.g. *637 ÷ 3*) • exchange of hundreds for tens only, with an overall remainder (e.g. *727 ÷ 4*) • exchange of tens only with (e.g. *563 ÷ 5*) or without (e.g. *570 ÷ 5*) an overall remainder • exchange of both hundreds and tens with (e.g. *963 ÷ 7*) or without (e.g. *612 ÷ 4*) an overall remainder. For now, keep to examples where the hundreds digit of the dividend is greater or equal to the divisor.
	Example – exchange of both hundreds and tens, without an overall remainder: 612 ÷ 4 = ?

Step 1 – write the divisor and dividend	
'Six hundred and twelve divided by four.'	$4\overline{)\,6\quad 1\quad 2}$
Step 2 – sharing the hundreds…	
	$\begin{array}{l} \quad\ 1 \\ 4\overline{)\,6\quad 1\quad 2} \end{array}$
6 hundreds ÷ 4 = 1 hundred r 2 hundreds	*'Write "1" in the hundreds column…'*

Step 3 –…and exchanging	
	$\begin{array}{r} 1\phantom{\,^{2}1\;\;2} \\ 4\overline{)6\,^{2}1\;\;2} \end{array}$
2 hundreds = 20 tens *'…and write "2" to the left of the tens digit of the dividend to make twenty-one tens.'*	
Step 4 – sharing the tens…	
	$\begin{array}{r} 1\;\;5 \\ 4\overline{)6\,^{2}1\;\;2} \end{array}$
21 tens ÷ 4 = 5 tens r 1 ten	*'Write "5" in the tens column…'*
Step 5 –… and exchanging	
	$\begin{array}{r} 1\;\;5\phantom{\;\;^{1}2} \\ 4\overline{)6\,^{2}1\;\;^{1}2} \end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*	

Step 6 – sharing the ones	
12 ones ÷ 4 = 3 ones *'Write "3" in the ones column.'*	$\begin{array}{r} 1\;\;5\;\;3 \\ 4\overline{)6\,^{2}1\;\;^{1}2} \end{array}$

4:7

Now spend some time working through a problem where the hundreds digit of the dividend is smaller than the divisor (e.g. *215 ÷ 5*). A common error that children make is to transfer an incorrect 'remainder' to the tens, equal to the difference between the divisor and the hundreds digit of the dividend, rather than exchanging all of the hundreds into tens:

$$\begin{array}{r|l} & 0\;\;6\;\;3 \\ \hline 5) & 2\;\;{}^{3}1\;\;{}^{1}5 \end{array}\;\; ✗ \qquad\qquad \begin{array}{r|l} & 0\;\;4\;\;3 \\ \hline 5) & 2\;\;{}^{2}1\;\;{}^{1}5 \end{array}\;\; ✓$$

This follows on from what children learned, in step *2:8*, regarding calculations where the ones digit of the dividend is smaller than the divisor (*83 ÷ 4*).

Emphasise the importance of writing a '0' in the hundreds column then exchanging the hundreds for tens.

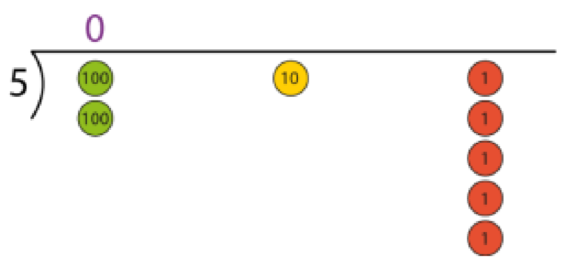
Note that, in the representations below, the counters are only illustrated for sharing and exchange of the hundreds. However, you should use counters throughout the calculation.

Work through a variety of similar calculations (e.g. *488 ÷ 8*), gradually removing the scaffolding of place-value counters until children are confident.

Hundreds digit of the dividend is smaller than the divisor:

215 ÷ 5 = ?

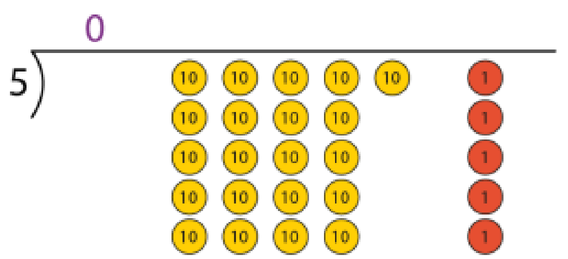
Sharing the hundreds



$$\begin{array}{r|l} & 0 \\ \hline 5) & 2\;\;\;1\;\;\;5 \end{array}$$

2 hundreds ÷ 5 = 0 hundreds r 2 hundreds — *'Write "0" in the hundreds column…'*

Exchanging



$$\begin{array}{r|l} & 0 \\ \hline 5) & 2\;\;{}^{2}1\;\;\;5 \end{array}$$

2 hundreds = 20 tens

'…and write "2" to the left of the tens digit of the dividend to make twenty-one tens.'

<table>
<tr><th>Skill: Divide 4-digits by 1-digit (grouping)</th><th>Year: 5</th></tr>
<tr><td>
<table>
<tr><td></td><td>4</td><td>2</td><td>6</td><td>6</td></tr>
<tr><td>2</td><td>8</td><td>5</td><td>$^{1}3$</td><td>$^{1}2$</td></tr>
</table>
Refer to Year 4 division method and apply with 4 digit by 1 digit division.
Use place value counters due to the size of numbers involved.
</td><td>
Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.
Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.
</td></tr>
</table>

Skill: Divide multi digits by 2-digits (short division)	Year: 6

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)	Year: 6
Please refer to the model on the following page.	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.

Teaching point 3:

When there is a remainder, the result can be expressed as a whole-number quotient and a whole-number remainder, as a whole-number quotient and a proper-fraction remainder, or as a decimal-fraction quotient.

Steps in learning

	Guidance	Representations
3:1	Now that children can confidently divide by two-digit divisors, explore the different ways to express the outcome of calculations with remainders, including: • as a whole-number quotient and a whole-number remainder (e.g. *730 ÷ 25 = 29 r 5*) • as a decimal fraction (e.g. *730 ÷ 25 = 29.2*) • as a whole-number quotient and a proper-fraction $\left(\text{e.g. } 730 \div 25 = 29\frac{1}{5}\right)$. When introducing the three options, the dividend and divisor should be kept the same, as exemplified opposite, so that the focus is on the difference in the way the remainder is expressed. Note that, in steps *3:1–3:4*, calculations are selected such that when the answer is expressed as a decimal fraction, only one decimal place is required. Before beginning, you may wish to recap learning from segment *2.12 Division with remainders, Teaching point 3* regarding the careful interpretation of calculations with remainders in connection with contextual problems. Then apply this to a problem solved using long division as illustrated opposite (children already learnt to express remainders as, for example, *29 r 5* in *Teaching point 2*).	Long division – remainder expressed as a whole number: *'Sue has seven hundred and thirty books. She packs them into boxes of twenty-five.'* • *'How many full boxes are there?'* • *'How many boxes does she need to pack all of the books?'* • *'How many books are not in a full box?'* 730 ÷ 25 = ? $\begin{array}{r@{\,}l} & \;2\;\;9\;\;\text{r}\,5 \\ 25 \overline{)} & 7\;\;3\;\;0 \\ & 5\;\;0 \\ \hline & 2\;\;3\;\;0 \\ & 2\;\;2\;\;5 \\ \hline & 0\;\;0\;\;5 \end{array}$ $\begin{array}{\|r\|r\|} \hline & \times 25 \\ \hline 1 & 25 \\ 2 & 50 \\ 4 & 100 \\ 5 & 125 \\ 8 & 200 \\ 10 & 250 \\ \hline \end{array}$ • So, 730 ÷ 25 = 29 r 5 Full boxes: 29 Boxes needed: 30 Books not in a full box: 5 Short division alternative: $\begin{array}{r@{\,}l} & 0\;\;\;2\;\;\;\;9\;\;\text{r}\,5 \\ 25 \overline{)} & 7\;\;{}^{7}3\;\;{}^{23}0 \end{array}$

<table>
<tr>
<td>3:2</td>
<td>Now present a context with the same dividend and divisor as in step 3:1, but for which it would be more appropriate to express the result as a decimal fraction; for example: 'Dinesh sells twenty-five jumpers, each for the same amount, and makes a total of £730. How much did each jumper sell for?'

Discuss why it isn't appropriate to use the answer from step 3:1, because it only tells us that each jumper costs £29 'and a bit' and we need to know the exact price. Note that, in this particular case, expressing the remainder as a fraction, as in the next step, could be used to convert the answer, if children know that one-fifth of £1 is 20 p.

Demonstrate how to continue the application of the long division algorithm to work out the value of the digit after the decimal point. Draw attention to the alignment of the decimal point, and exchange of 5 ones for 50 tenths. As usual, describe the whole-number part of the calculation with unitising until you reach the remainder (Step 1 opposite); then continue the calculation, describing the introduction of the decimal point (Step 2 opposite) and unitising in tenths (Steps 3 and 4 on the next page).

A common error is for children to write the remainder in the tenths column (resulting in an incorrect answer of 29.5 in this example), without taking into account that the remainder (5) needs to be divided by the divisor (25).</td>
<td>Long division – remainder converted to decimal fraction:

'Dinesh sells twenty-five jumpers, each for the same amount, and makes a total of £730. How much did each jumper sell for?'

730 ÷ 25 = ?

Step 1 – calculate the whole-number quotient:

$$\begin{array}{r} 29 \\ 25\overline{)730} \\ \underline{50} \\ 230 \\ \underline{225} \\ 5 \end{array}$$
• 73 tens ÷ 25 = 2 tens and a remainder
'Write "2" in the tens column of the answer line and write "50" underneath the "73".'
• 73 tens – 50 tens = 23 tens
'Write "23" underneath the "50".'
• 23 tens = 230 ones
'Write "0" after the "23".'
• 230 ones ÷ 25 = 9 ones and a remainder
'Write "9" in the ones column of the answer line and write "225" underneath the "230".'
• 230 ones – 225 ones = 5 ones
'Write "5" underneath the "225".'

Step 2 – introduce the decimal point:

$$\begin{array}{r} 29. \\ 25\overline{)730.0} \\ \underline{50} \\ 230 \\ \underline{225} \\ 5 \end{array}$$
• 'There is a remainder. To represent this as a decimal fraction, first write a decimal point after the ones digit of both the dividend and the quotient. Write a place-holder zero in the tenths column of the dividend.'</td>
</tr>
</table>

		Step 3 – continue: $\begin{array}{r} 2\;9\;. \\ 25\overline{)7\;3\;0\;.\;0} \\ \underline{5\;0} \\ 2\;3\;0 \\ \underline{2\;2\;5} \\ 5\;\;0 \end{array}$ • 5 ones = 50 tenths *'Write "0" after the "5".* Step 4 – complete the calculation: $\begin{array}{r} 2\;9\;.\;2 \\ 25\overline{)7\;3\;0\;.\;0} \\ \underline{5\;0} \\ 2\;3\;0 \\ \underline{2\;2\;5} \\ 5\;\;0 \\ \underline{5\;\;0} \\ 0 \end{array}$ • 50 tenths ÷ 25 = 2 tenths *'Write "2" in the tenths column of the answer line and write "50" underneath the "50".'* • 50 tenths – 50 tenths = 0 tenths *'Write "0" underneath the "50".'* • So, 730 ÷ 25 = 29.2 • *'Each jumper sold for £29.20.'* Short division alternative: $\begin{array}{r} 0\quad 2\quad 9\;.\;2 \\ 25\overline{)7\;\;{}^{7}3\;\;{}^{23}0\;.\;{}^{5}0} \end{array}$
3:3	Keeping the same dividend and divisor, explore a context for which it would be appropriate to express the result as a proper fraction. Again, use the usual descriptive language to explain the calculation until you reach the remainder; then describe as exemplified on the next page.	

<table>
<tr>
<td></td>
<td>When providing the final answer to the question, it is important for children to express the fractional remainder in its simplest form; children learnt to simplify fractions in Spine 3: Fractions, segment 3.7.

Note that in the example opposite, it would also be appropriate to say that Saveeta travels 29.2 times the distance that Lily travels.</td>
<td>Long division – remainder converted to proper fraction:

'Saveeta travels 730 km to visit her grandma. Lily travels 25 km to visit her grandma. How many times Lily's distance does Saveeta travel?'

$730 \div 25 = ?$

$$\begin{array}{r} 2\;9\;\frac{5}{25} \\ 25\overline{)7\;3\;0} \\ \underline{5\;0} \\ 2\;3\;0 \\ \underline{2\;2\;5} \\ 5 \end{array}$$
- 'There is a remainder. To represent this as a fraction, write the remainder as the numerator and the divisor as the denominator.'

$5 \div 25 = \frac{5}{25}$

'Write $\frac{5}{25}$ after the ones column of the answer line.'

- 'Simplify the fraction.'

$\frac{5}{25} = \frac{1}{5}$

- So, $730 \div 25 = 29\frac{1}{5}$

- 'Saveeta travels twenty-nine and one-fifth times the distance that Lily travels.'

Short division alternative:

$$\begin{array}{r} 0\quad 2\quad 9\;\frac{5}{25} \\ 25\overline{)7\quad {}^{7}3\quad {}^{23}0} \end{array}$$</td>
</tr>
<tr>
<td>3:4</td>
<td colspan="2">Using a different calculation, show three algorithms alongside each other, representing the three different ways of expressing the remainder. Then discuss, for a variety of contexts, which representation is appropriate. Note that there isn't necessarily one 'correct' answer; in many cases both the decimal fraction and the proper fraction representation of the remainder are appropriate.</td>
</tr>
</table>

$354 \div 15 = ?$

$\begin{array}{r} 2\ 3\ \ r9 \\ 15\overline{)3\ 5\ 4} \\ \underline{3\ 0} \\ 5\ 4 \\ \underline{4\ 5} \\ 9 \end{array}$	$\begin{array}{r} 2\ 3\ \frac{9}{15} \\ 15\overline{)3\ 5\ 4} \\ \underline{3\ 0} \\ 5\ 4 \\ \underline{4\ 5} \\ 9 \end{array}$	$\begin{array}{r} 2\ 3\ .\ 6 \\ 15\overline{)3\ 5\ 4\ .\ 0} \\ \underline{3\ 0} \\ 5\ 4 \\ \underline{4\ 5} \\ 9\ \ 0 \\ \underline{9\ \ 0} \\ 0 \end{array}$
So, $354 \div 15 = 23$ r 9	$\frac{9}{15} = \frac{3}{5}$ So, $354 \div 15 = 23\frac{3}{5}$	So, $354 \div 15 = 23.6$

Possible contexts:

- *'Robert raised £354 for charity. Harrison raised £15. How many times as much did Robert raise compared to Harrison?'*
- *'Debo has cut 354 m of rope into fifteen equal lengths for climbers to use. How long is each piece?'*
- *'Rachel has 354 beads. She uses fifteen to make a bracelet.'*
  - *'How many bracelets can she make?'*
  - *'How many beads does she have left over?'*
- *'Some children wash cars to earn some money for their holidays. There are fifteen children and they earn £354 altogether. How much does each child get if they share the money equally?'*
- *'One bin holds 15 kg of rubbish. During a school week 354 kg of rubbish is collected. How many bins have been filled?'*

3:5

Now that the children have an efficient method to divide, work through a calculation in which the quotient has two decimal places, beginning with cases that have '25' or '75' after the decimal point of the quotient (e.g. *105 ÷ 20 = 5.25*). Use similar language to describe the stages of the calculation as that exemplified in steps *3:2* and *3:3*.

Then work through some examples that have answers with two decimal places but that don't result in '.25' or '.75', e.g. *141 ÷ 60* and *236 ÷ 50*.

$105 \div 20 = ?$

<table>
<tr>
<td>

$$\begin{array}{r} 5\tfrac{5}{20} \\ 20\overline{)1\;0\;5} \\ \underline{1\;0\;0} \\ 5 \end{array}$$

- 'There is a remainder. To represent this as a fraction, write the remainder as the numerator and the divisor as the denominator.'
- $5 \div 20 = \frac{5}{20}$
  'Write "$\frac{5}{20}$" after the ones column of the answer line.'
- 'Simplify the fraction.'
  $\frac{5}{20} = \frac{1}{4}$
- So, $105 \div 20 = 5\frac{1}{4}$

</td>
<td>

$$\begin{array}{r} 5\,.\,2\;5 \\ 20\overline{)1\;0\;5\,.\,0\;0} \\ \underline{1\;0\;0} \\ 5\;0 \\ \underline{4\;0} \\ 1\;0\;0 \\ \underline{1\;0\;0} \\ 0 \end{array}$$

- 'There is a remainder. To represent this as a decimal fraction, first write a decimal point after the ones digit of both the dividend and the quotient.'
- 'Write a place-holder zero in the tenths column of the dividend.'
- 5 ones = 50 tenths
  'Write "0" after the "5".'
- 50 tenths ÷ 20 = 2 tenths and a remainder
  'Write "2" in the tenths column of the answer line and write "40" underneath the "50".'
- 50 tenths − 40 tenths = 10 tenths
  'Write "10" underneath the "50".'
- 'Write a place-holder zero in the hundredths column of the dividend.'
- 10 tenths = 100 hundredths
  'Write "0" after the "10".'
- 100 hundredths ÷ 20 = 5 hundredths
  'Write "5" in the hundredths column of the answer line and write "100" underneath the "100".'
- 100 hundredths − 100 hundredths = 0 hundredths
  'Write "0" underneath the "100".'
- So, $105 \div 20 = 5.25$

</td>
</tr>
</table>

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor